NAME:

NetID:

MATH 285 G1 Exam 1 (A) **February 17, 2016** **Instructor: Pascaleff**

INSTRUCTIONS:

- Do all work on these sheets.
- Show all work.
- No notes, books, calculators, or other electronic devices are permitted.

Problem	Possible	Actual
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20 points) Consider the differential equation

$$\frac{dy}{dx} = (x+1)y$$

(a) (10 points) Draw a slope field for this equation.

(b) (10 points) Given the initial condition $y(1) = 1$, use Euler's method with two steps to approximate $y(1.2)$.

2. (20 points) Let $y(x)$ be a solution of the initial value problem

$$\frac{dy}{dx} = 1 + y^2, \quad y(0) = 3$$

Starting from $y_0(x) = 3$, compute the first and second Picard approximations $y_1(x)$ and $y_2(x)$, and use $y_2(x)$ to estimate $y(0.1)$.

3. (20 points) Find the general solution of

$$\frac{dy}{dx} = e^{2x} - 3y$$

4. (20 points) Let $P(t)$ be denote a population of fish in a lake. This population is governed by the differential equation

$$\frac{dP}{dt} = P(100 - P) - 200$$

(a) (10 points) Find the equilibrium solutions, and determine whether each is stable or unstable.

(b) (10 points) Draw a qualitative plot of the solutions of this differential equation.

5. (20 points) A metal ball has been heated to $500°C$. It is placed into a bath of water at $30°C$. After 5 seconds, it has cooled to a temperature of $200°C$.

   Suppose now that the metal ball is cooled to $0°C$, and again placed into a bath of water at $30°C$. How long will it take to reach a temperature of $20°C$? Your answer does not need to be simplified.

   In both situations, the process is governed by Newton's law of cooling:

$$\frac{dT}{dt} = -k(T - A)$$

   where A is the temperature of the water, and k is a constant.

This page is for work that doesn't fit on the other pages.