Math 285 — Midterm 1 practice

Total points: **100**. Please show your work and explain all answers. Calculators, computers, books and notes are **not** allowed. Suggestion: even if you cannot complete a problem, write out the part of the solution you know. You can get partial credit for it.

1. **[20 points]** Solve the following initial value problem for $y(x)$:

$$y'' + 2y' + 6y = 0; \qquad y(0) = 0; \qquad y'(0) = 2$$

2. [20 points] Solve the following initial value problem for $u(t)$:

$$\frac{\mathrm{d}^3 u}{\mathrm{d}t^3} = -e^t; \qquad u(0) = 0; \qquad u'(0) = 0; \qquad u''(0) = 2$$

3. [**20 points**] The evolution of the population $P(t)$ is described by the equation $\mathrm{d}P/\mathrm{d}t = P^2 - 5P + 6$. Sketch the corresponding slope field and indicate on it the equilibrium solutions and their stability. Consider then the population with initial condition $P(0) = 4$ and draw its evolution. What happens to this polulation as t increases? Verify your answer by calculating the exact solution for the given initial condition.

4. [20 points] Solve the following initial value problem for $x(t)$:

$$\frac{\mathrm{d}x}{\mathrm{d}t} + tx = te^{-t^2/2}; \qquad x(0) = -1$$

5. [**20 points**] Find the general solution for the following ODE for $y(x)$:

$$2 + \frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{2x + y}$$