Math 285 — Midterm 1 practice

Total points: 100. Please show your work and explain all answers. Calculators, computers, books and notes are not allowed. Suggestion: even if you cannot complete a problem, write out the part of the solution you know. You can get partial credit for it.

1. [20 points] Solve the following initial value problem for $y(x)$:

$$y'' + 2y' + 6y = 0; \quad y(0) = 0; \quad y'(0) = 2$$
2. [20 points] Solve the following initial value problem for $u(t)$:

$$\frac{d^3u}{dt^3} = -e^t; \quad u(0) = 0; \quad u'(0) = 0; \quad u''(0) = 2$$
3. [20 points] The evolution of the population $P(t)$ is described by the equation $\frac{dP}{dt} = P^2 - 5P + 6$. Sketch the corresponding slope field and indicate on it the equilibrium solutions and their stability. Consider then the population with initial condition $P(0) = 4$ and draw its evolution. What happens to this population as $t$ increases? Verify your answer by calculating the exact solution for the given initial condition.
4. [20 points] Solve the following initial value problem for $x(t)$:

$$\frac{dx}{dt} + tx = te^{-t^2/2}; \quad x(0) = -1$$
5. [20 points] Find the general solution for the following ODE for \( y(x) \):

\[
2 + \frac{dy}{dx} = \sqrt{2x + y}
\]