Math 285 — Midterm 3 practice

Total points: **100**. Please explain all answers. Calculators, computers, books and notes are **not** allowed. Suggestion: even if you cannot complete a problem, write out the part of the solution you know. You can get partial credit for it.

1. [**25 points**] Calculate (so don't give me a memorized answer for) the Sine Fouries series expansion for $f(t) = t^2$ in $0 < t < 2$.

2. [25 points] Find a solution $y(x)$ for $0 \leq x \leq \pi$ to the following boundary value problem:

$$y'' + \frac{1}{25} y = \sum_{n=1}^{\infty} \frac{1}{n} \sin(nx)$$

$$y(0) = y(\pi) = 0$$

Do you need a complementary solution $y_c(x)$?

3. [**25 points**] Using separation of variables, solve the following diffusion problem for $y(x, t)$ (In other words, don't use the formula for the solution of the diffusion equation. Derive your result)

$$y_t = 3\, y_{xx}$$
$$y_x(0, t) = y_x(2, t) = 0$$
$$y(x, 0) = 4 + 3\cos(\pi x)$$

4. [25 points] Solve the following PDE for $y(x,t)$ and the given auxiliary conditions for $0 \le x \le 1$ and $t \ge 0$ (Derive your result. Do not use a formula for the solution of the wave equation)

$$y_{tt} = 4\,y_{xx}$$

$$y(0,t) = y(1,t) = 0$$

$$y(x,0) = 0$$

$$y_t(x,0) = \sin(\pi x) - \sin(3\pi x)$$

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