Math 221/AL1 Final Exam

UIUC, December 15, 2012

Name ______________________________

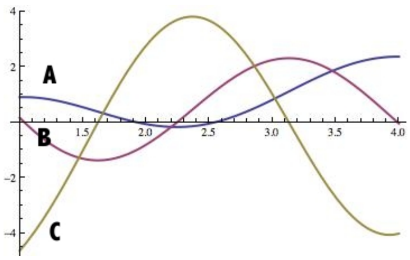
WF Section (circle yours)

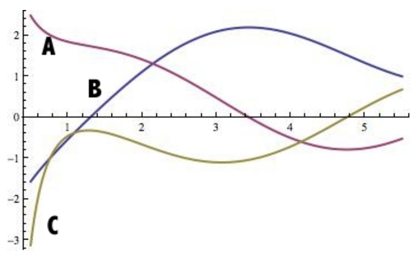
8-8:50 (ADA) Dong Dong	9-9:50 (ADB) Dong Dong	10-10:50 (ADC) Mahmood Etedadi Aliabadi	11-11:50 (ADD) Amelia Tebbe
12-12:50 (ADE) Fan Huang	1-1:50 (ADF) Fan Huang	2-2:50 (ADG) Jordan Hasler	3-3:50 (ADH) Jordan Hasler

Instructions. Please provide your answers in the space provided—using the back of sheets if necessary. No calculators, electronics, friends or psychic readings are to be used while taking this test. If you are unclear about a question—please raise your hand and ask.

Ques.	Pts.		Ques.	Pts.		Ques.	Pts.		Ques.	Pts.	
1	6		7	10		13	12		18	10	
2	10		8	12		14	20		19	10	
3	18		9	10		15	12				
4	8		10	10		16	8				
5	10		11	8		17	8				
6	8		12	10							
Tot	60		Tot	60		Tot	60				

1. (*6 points*) Each of the following has the graph of a function, its derivative and its second derivative. Identify which one is which.





$f(x) =$ ____ $\qquad$ $f(x) =$ ____

$f'(x) =$ ____ $\qquad$ $f'(x) =$ ____

$f''(x) =$ ____ $\qquad$ $f''(x) =$ ____

2. (*10 points*)

Use the *definition* of the derivative to show

$$\text{if} \quad f(x) = x^2 - x + 1 \qquad \text{then} \qquad f'(x) = 2x - 1$$

3. (*18 points, 3/3/3/3/3/3*)

Compute the following derivatives using any methods we have covered in class.

(a) $\dfrac{4x}{\sqrt[5]{x}} + \dfrac{e^x}{2}$

(b) $\ln x \sin x$

(c) $\dfrac{\tan x}{x^3 + 1}$

(d) $(8 - 5x^4)^8$

(e) $3^{5x+\cos x}$

(f) $\sqrt[3]{x + \sec x^2}$

4. (*8 points*)

   Find an equation of the tangent line to the curve at the the given point.

$$y = 2x^5 - 3x^3 + 2, \qquad (1, 1)$$

5. (*10 points*) Find the horizontal and vertical asymptotes of the curve.

   (a) $y = \dfrac{-x^2 + 4x - 3}{x^2 - 6x + 5}$

   (b) $y = \dfrac{2^x}{2^x - 5}$

6. (*8 points*) Using the methods of this course, explain why the following polynomial has a unique real root.

$$y = x^5 + x^3 + 3x - 4$$

7. (*10 points*) Find the absolute maximum and absolute minimum values of $f(x) = xe^{5x}$ on $[-1, 1]$.

8. (*12 points, 2/2/3/5*) Determine the limit if it exists. Be sure to show your reasoning. No work, no credit.

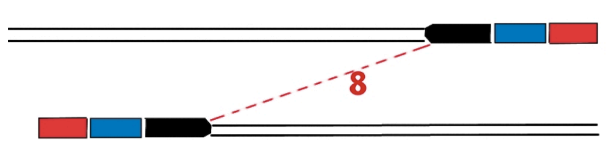
(a) $\displaystyle\lim_{x\to 0} \frac{\tan 3x}{x + \sin 5x}$.

(b) $\displaystyle\lim_{x\to 0} \frac{\sin^2 x - x}{e^{2x}}$.

(c) $\displaystyle\lim_{x\to 0^+} 3x \ln 4x$

(d) $\displaystyle\lim_{x\to 0^+} x^{\sqrt[3]{x+1}}$

9. (*10 points*) Two trains are on parallel tracks that are 2 miles apart. If both trains are traveling at 40 miles per hour, how fast is the distance between the trains decreasing when they are 8 miles apart from one another?



10. (*10 points*) Determine the equation of the tangent line to the graph of

$$xe^y + 5ye^x = 1$$

at the point $(1, 0)$.

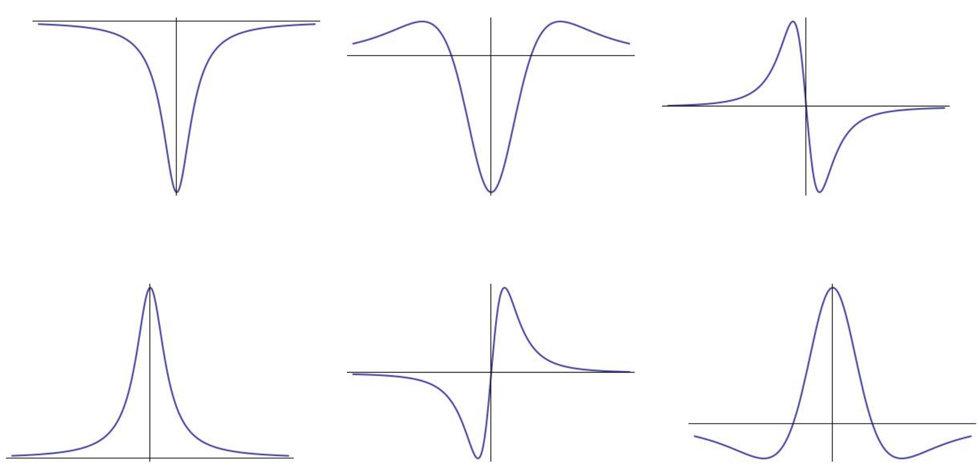
11. (*8 points, 2/2/2/2*) Consider the function $f(x) = \dfrac{1}{1+2x^2}$.

(a) How does f behave as $x \to \pm\infty$?

(b) Where are the critical points of f? Where is f increasing, decreasing?

(c) Where is $f'' = 0$? Where is f concave up, down?

(d) Circle the graph that best fits the data you have determined?



12. (*10 points*) According to US postal regulations, a carton is “oversized” if the sum of its height and girth (the perimeter of its base) exceeds 108 in. Find the dimensions of a carton with <u>square</u> base that is not oversized and has maximum volume.

13. (*12 points, 3/3/6*) This question concerns the definite integral

$$\int_0^4 3 - 2x\,dx$$

(a) Sketch the area represented by the integral and compute it using geometry.

(b) Solve the integral using the fundamental theorem of calculus.

(c) Compute the integral by computing the limit of the Riemannian sums as the partitions go to ∞.

14. (*20 points*) Compute the indefinite integral.

(a) $\displaystyle\int x\sqrt[4]{2+x}\,dx$

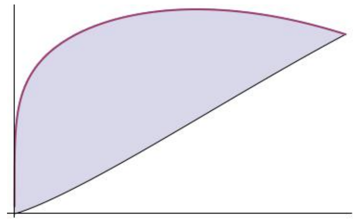
(b) $\displaystyle\int w\,3^{w^2+1}\,dw$

(c) $\displaystyle\int \frac{2t}{\sqrt{1+t^4}}\,dt$

(d) $\displaystyle\int \sec^2(3\theta)\tan^3(3\theta)\,d\theta$

(e) $\displaystyle\int \frac{1-2x}{\sqrt{1-x^2}}\,dx$

15. (*12 points, 4/4/4*) The shaded region is bounded by the curves $y = \sqrt[5]{x} \sin x$ and $y = \sqrt[5]{x} \cos x$

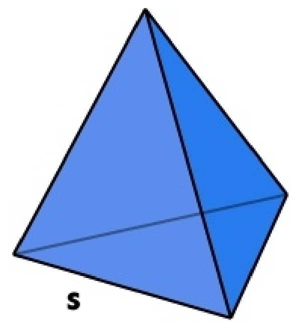


(a) Give an integral (do not evaluate) for the area of the region.

(b) Give an integral (do not evaluate) for the volume obtained if this region is rotated about the line $y = 4$.

(c) Give an integral (do not evaluate) for the volume obtained if this region is rotated about the line $x = 3$.

16. (*8 points*) Use integration to compute the volume of a regular tetrahedron whose faces are equilateral triangles of side length s. Hint: The height is: $s\sqrt{\dfrac{2}{3}}$.



17. (*8 points*)

Determine the value x where the maximum of the function $g(x)$ occurs if

$$g(x) = \int_{x}^{x+3} e^{-t^2}\, dt$$

18. (*10 points*) Indicate which Mathematical Theorem *best fits* the statement.

    Ferm Fermat's Theorem

    IVT Intermediate Value Theorem

    EVT Extreme Value Theorem

    MVT Mean Value Theorem

    FTC Fundamental Theorem of Calculus

    _____ There was a maximum temperature in the last 24 hours.

    _____ A ball tossed in the air will be at its highest point when it stops moving for an instant.

    _____ On a trip, the distance from the end compared to the beginning is measured by the area under the graph of the velocity.

    _____ A fish above the water surface will make a splash before it is again underwater.

    _____ If you travel 90 miles in 2 hours, at some time you were traveling 45 miles per hour.

19. (*10 points*)

    The function $H(x)$ is defined on $[0, \frac{\pi}{2}]$ by

    $$H(x) = \int_{\cos x}^{\sin x} \sqrt{1-t^2}\, dt$$

    Show that $H'(x) = 0$ and use this to determine $H(.2)$ after computing the value of H at $\dfrac{\pi}{4}$.