Math 231 ADF. Final Exam. Dec. 12, 2011. 8-11 am.

Name: ________________________ **Section Code:** [ ][ ][ ]

Three points will be deducted if these instructions are not followed.

1. Write your full name legibly above.
2. Write your section code from the chart below (two letters and one number) in the boxes above.
3. Code your name, netid, and test form A correctly on the scantron form.

Ackermann, Colleen	**DD2:** 11 am, 140 Henry **FD2:** 10 am, 2 Illini	Pan, Ziying	**FD9:** 9 am, 143 Henry **AD8:** 10 am, 345 Altgeld
Addabbo, Darlayne	**DD3:** 12 pm, 143 Henry **AD4:** 1pm, 147 Altgeld	Park, Hyunchul	**FD4:** 1 pm, 1 Illini
Anders, Katherine	**DD7:** 9 am, 159 Altgeld	Reiniger, Benjamin	**AD7:** 3 pm, 173 Altgeld
Collier, Brian	**FD1:** 4 pm, 341 Altgeld	Rochford, Austin	**DD8:** 1 pm, 447 Altgeld
Freidin, Brian	**DD4:** 1 pm, 142 Henry	Sanchez, Mychael	**AD6:** 11 am, 341 Altgeld
Johnson, Matt	**AD2:** 9 am, 140 Henry	Sewry, Julie	**AD9:** 12 pm, 168 Everitt
Keyvan, Chanel	**FD6:** 2 pm, 7 Illini	Shahkarami, Eric	**DD5:** 2 pm, 143 Henry
Kim, Eunmi	**AD3:** 10 am, 147 Altgeld	Shahkarami, Sean	**DD1:** 4 pm, 441 Altgeld **FD5:** 5 pm, 341 Altgeld
Koss, Adam	**FD7:** 3 pm, 143 Henry	Sriponpaew, Boonyong	**AD1:** 8 am, 142 Henry **DD9:** 9 am, 445 Altgeld
Liang, Jian	**FD3:** 12 pm, 162 Noyes **DD0:** 1 pm, 57 Everitt	Yeager, Elyse	**AD5:** 2 pm, 145 Altgeld
Nelson, Peter	**FD8:** 9 am, 2 Illini **DD6:** 3 pm, 145 Altgeld		

- **Multiple choice answers must be marked on scantron form.**
- There are 150 total points (70 multiple choice and 80 free response).
- No written materials of any kind allowed. No scratch paper unless provided by proctors.
- No phones, calculators, iPods or electronic devices of any kind are allowed for ANY reason, including checking the time (you may use a simple wristwatch).
- Do not turn this page until instructed to.
- There are several different versions of this exam.

Violations of academic integrity (in other words, cheating) will be taken extremely seriously.

Free response scores–for graders only

1	2	3	4	5	Total
18 points	18 points	18 points	18 points	8 points	80

Multiple choice. Mark answers on scantron form. Test form A.

Multiple choice problems are worth 5 points each unless otherwise noted.

Trig identities: $\sin^2 x + \cos^2 x = 1$, $\tan^2 x + 1 = \sec^2 x$, $\sin 2x = 2\sin x \cos x$, $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

1. Evaluate $\displaystyle\int_1^{e^2} \ln x \, dx$.

(A) 1

(B) e

(C) $1 + 2e$

(D) $1 + e^2$

(E) $e^2 - 1$

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2. Evaluate $\displaystyle\int_0^1 x e^{2x} \, dx$.

(A) $2e^2 - 2$

(B) $e^2 + 1$

(C) $e^2 + \frac{3}{2}$

(D) $\frac{1}{4}e^2 + 1$

(E) $\frac{1}{4}(e^2 + 1)$

3. Evaluate $\displaystyle\int \frac{9x-1}{x^2-1}\,dx$.

(A) $4\ln|x-1| + 5\ln|x+1| + C$

(B) $5\ln|x-1| + 4\ln|x+1| + C$

(C) $6\ln|x-1| + 3\ln|x+1| + C$

(D) $4\ln|x-1| - 5\ln|x+1| + C$

(E) $6\ln|x-1| - 3\ln|x+1| + C$

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4. After making the correct trig substitution, which does the following integral become: $\displaystyle\int \frac{1}{x^2\sqrt{9+x^2}}\,dx$.

(A) $\frac{1}{9}\displaystyle\int \frac{\cos^2\theta}{\sin\theta}\,d\theta$

(B) $\frac{1}{9}\displaystyle\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$

(C) $\frac{1}{9}\displaystyle\int \cos^2\theta\sin^2\theta\,d\theta$

(D) $\frac{1}{9}\displaystyle\int \frac{\cos^3\theta}{\sin^2\theta}\,d\theta$

(E) $\frac{1}{9}\displaystyle\int \cos\theta\sin\theta\,d\theta$

5. Find all values of p for which the following integral converges: $\displaystyle\int_2^\infty \frac{1}{x(\ln x)^p}\,dx.$

(A) $p < 1$
(B) $p \le 1$
(C) $p \ge 0$
(D) $p > 1$
(E) $p \ge 1$

6. Find the length of the curve $y = 1 + \frac{2}{3}x^{3/2}, \quad 0 \le x \le 1.$

(A) $\frac{2}{3}\left(2\sqrt{2} - 1\right)$
(B) $2\sqrt{3} - \frac{4\sqrt{2}}{3}$
(C) $2/3$
(D) $\sqrt{3} - \frac{1}{3}$
(E) $\sqrt{3} - \frac{1}{3}$

Problems on this page are worth 2 points each

Mark C if the integral converges, and D if the integral diverges

7. $\displaystyle\int_0^\infty \frac{x^2}{\sqrt{x^7+14}}\,dx$

8. $\displaystyle\int_1^\infty \frac{e^{-x}+x}{(x+2)^2}\,dx$

9. $\displaystyle\int_0^1 \frac{1}{|x|^{\frac{1}{3}}}\,dx$

Mark C if the sequence $\{a_n\}$ converges and D if it diverges

10. $a_n = \dfrac{n^2 + n\sin(n)}{n^2\ln(n)+n}$

11. $a_n = \dfrac{(\ln n)^{50}}{\sqrt{n}}$

12. $a_n = \dfrac{n+(-1)^n n^2}{n^2+1}$

Mark A if the series converges absolutely.
Mark C if the series converges conditionally.
Mark D if the series diverges.

13. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n!}$

14. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n^2}{n^3+7}$

15. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cos(1/n)$

16. $\displaystyle\sum_{n=2}^{\infty} n^5 \left(\frac{-2}{3}\right)^n$

17. Find the sum of the series $\sum_{n=1}^{\infty} \frac{5^{2n}}{2^{5n+1}}$.

(A) 25/64

(B) 25/32

(C) 25/14

(D) 14/64

(E) 14/32

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18. Find the Maclaurin series for the function $\frac{1-\cos(x^2)}{x^2}$.

(A) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n-2}}{(2n)!}$

(B) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(4n)!}$

(C) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n)!}$

(D) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{4n}}{(4n)!}$

(E) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{4n-2}}{(2n)!}$

19. Find the equation for the polar curve $r = 4\sin\theta$ in rectangular coordinates.

(A) $x^2 + y^2 - 4 = 0$
(B) $x^2 + y^2 - 2x = 0$
(C) $x^2 + y^2 - 2y = 0$
(D) $x^2 + y^2 - 4x = 0$
(E) $x^2 + y^2 - 4y = 0$

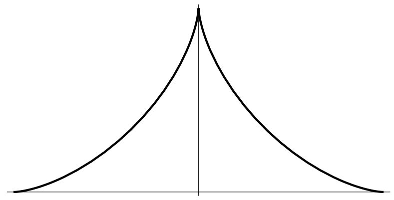
20. Find the total area enclosed by the polar curve $r = \sin(2\theta)$.

(A) π
(B) $\pi/2$
(C) $\pi + 1$
(D) $\pi/2 + 1$
(E) 1

Part II. Free response. Show your work, and circle your answers.

1. (18 Points) Consider the curve (shown below) defined by the parametric equations

$$x = \cos^3(t), \quad y = \sin^3(t), \quad 0 \le t \le \pi$$



(i) Set up **but do not evaluate** an integral which represents the area under the curve.

(ii) Set up **but do not evaluate** an integral which represents the length of the curve.

(iii) Evaluate the integral from the last part to find the actual length.

2. (18 Points)
(i) Give the Maclaurin series for the function $\cos x - 1$.

(ii) Use series to evaluate the limit $\lim_{x \to 0} \dfrac{\cos x - 1}{x^2 e^x}$.
No credit for using a different method.

(iii) Estimate $\int_0^1 (\cos x - 1)\, dx$ to within .001.
You do need not need to simplify your answer at all. But you must justify the accuracy.

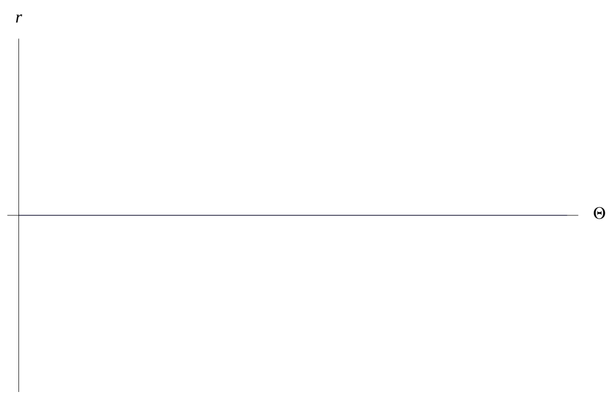
3. (18 Points) (i) Find the Taylor series for $f(x) = xe^x$ centered at 0.

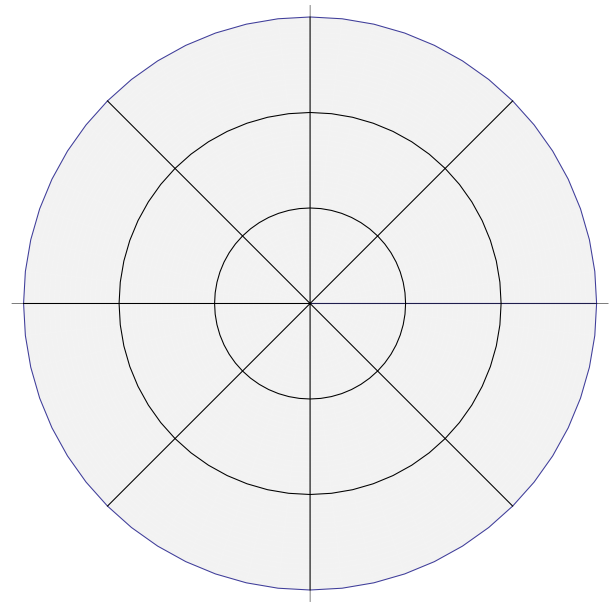
(ii) Find the degree two Taylor polynomial $T_2(x)$ for $f(x) = xe^x$ centered at 0.

(iii) $T_2(x)$ is used to approximate $f(x)$ in the range $0 \leq x \leq 1/2$. Estimate the maximum error in this approximation (you do not need to simplify your answer).

4. (18 Points)

(i) Make a careful sketch of the polar curve $r = 1 + \sqrt{2}\cos\theta$ on the polar axes. Begin by making a careful rectangular sketch on the axes to the left. Indicate all important features of your sketches, including all important angles.





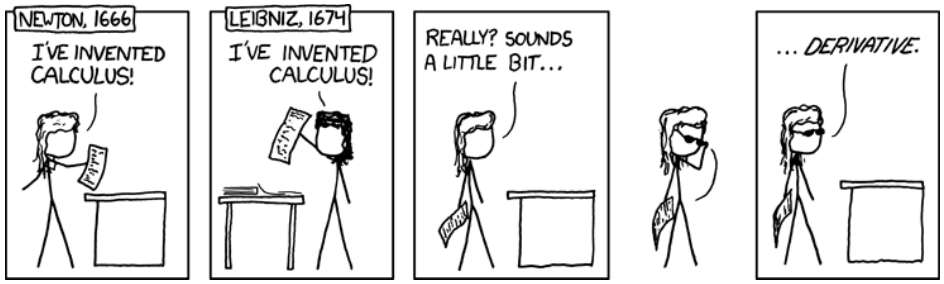
(ii) Find the slope of the tangent line to the polar curve at the point where $\theta = \pi/2$.

(iii) Set up and simplify **but do not evaluate** an integral which represents the total length of the polar curve.

5. (8 points) Consider the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^n}{n+1}$.

(i) Find the radius of convergence.

(ii) Find the interval of convergence.


NEWTON, 1666
I'VE INVENTED CALCULUS!
LEIBNIZ, 1674
I'VE INVENTED CALCULUS!
REALLY? SOUNDS A LITTLE BIT...
... DERIVATIVE.