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Math 285 — Final exam practice

Total points: **100**. Please show the work you did to get the answers. Calculators, computers, books and notes are **not** allowed. Suggestion: even if you cannot complete a problem, write out the part of the solution you know. You can get partial credit for it.

1. [10 points] Find the general solution of the following ODE for $y(x)$:

$$y^{(5)} - 4y^{(4)} + 4y^{(3)} = 0$$

(where $y^{(4)}$ means the fourth derivative of y)

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2. [10 points] Solve the following initial value problem for $y(x)$:

$$y'' - \frac{1}{x^2} = 0; \qquad y(1) = y'(1) = 0$$

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3. [15 points] Find the general solutions of the following ODE's for $y(x)$:

$$
\begin{array}{ll}
\text{a)} & y'' = 4y \\
\text{b)} & y' = y \sin(x) \\
\text{c)} & y'' + 2y = 0
\end{array}
$$

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4. [15 points] Find all eigenvalues and associated eigenfunctions for the following boundary value problem for $y(x)$:

$$y'' - 2y' + \lambda y = 0$$

$$y(0) = y(2) = 0$$

You may want to consider the substitution $y(x) = e^x g(x)$. (But your final answer has to be in terms of $y(x)$)

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5. [**10 points**] Calculate the Fourier series expansion for the following function of period 2:

$$f(t) = 2 + 2\,t^2 \qquad \text{for } -1 < t < 1$$

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6. [**15 points**] Find the solution $y(x,t)$ in $0 < x < 3$ and $t \geq 0$ for the following heat conduction problem:

$$2y_t = y_{xx}; \qquad y_x(0,t) = y_x(3,t) = 0; \qquad y(x,0) = g(x)$$

where $g(x)$ is a generic function. Derive your solution using separation of variables. Don't rely on a formula. What happens as $t \to +\infty$? Briefly discuss the physical meaning of this result.

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7. [10 points] Consider the following population equation for $P(t)$ (assume it makes sense to also consider negative values of P here). Sketch a slope field and indicate on it the equilibrium solutions and their stability. Then sketch the behaviour of the three solutions corresponding to the given initial conditions (which are not given at $t = 0$!).

$$\frac{dP}{dt} = 16P - 8P^2 + P^3$$

a) $P(1) = -1$

b) $P(1) = 1$

c) $P(1) = 5$

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8. [15 points] Consider the following ODE for $x(t)$:

$$x'' + a\,x' + 16x = \sin(4t)$$

where a is a constant. Find the general solution $x(t)$ for $a = 0$ and then for $a = 1$. Briefly discuss the effect of a on the long term behaviour of the solution.