1. Consider the vectors \( \mathbf{u}, \mathbf{v}, \mathbf{w} \) in \( \mathbb{R}^2 \) shown at right.

For each of the following, circle the best answer (2 points each)

(a) \( \mathbf{w} = \begin{bmatrix} \mathbf{u} + 2\mathbf{v} & 2\mathbf{u} + \mathbf{v} & -2\mathbf{u} + \mathbf{v} & -\mathbf{u} + 2\mathbf{v} & \mathbf{u} - 2\mathbf{v} & 2\mathbf{u} - \mathbf{v} \end{bmatrix} \)

(b) \( \text{proj}_\mathbf{u} \mathbf{v} = \begin{bmatrix} 2\mathbf{u} & \frac{1}{2} \mathbf{u} & 2\mathbf{v} & \frac{1}{2} \mathbf{v} & 2\mathbf{w} & \mathbf{w} + \mathbf{v} & -2\mathbf{v} \end{bmatrix} \)

(c) \( \mathbf{u} \cdot \mathbf{w} = \)

> 

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0.

2. Suppose \( f(x, y) \) has the contour plot below right, with points labelled. Circle the best answer to each of the following questions: (1 point each)

(a) \( f(c) \) is: positive negative 0

(b) \( \frac{\partial f}{\partial x} (b) \) is: positive negative 0

(c) \( \frac{\partial^2 f}{\partial y^2} (d) \) is: positive negative 0

(d) Circle one:

\( \frac{\partial f}{\partial x} (a) > \frac{\partial f}{\partial x} (c) \) 
\( \frac{\partial f}{\partial x} (a) < \frac{\partial f}{\partial x} (c) \)
3. Circle the equation for the quadric surface shown at right.  (2 points)

1. \(x^2 - y^2 + z^2 = 1\)
2. \(-x^2 + y^2 + z^2 = 1\)
3. \(x^2 + y^2 - z^2 = -1\)
4. \(x - y^2 + z = 1\)
5. \(x^2 - y^2 - z^2 = 1\)

4. For each function \(f(x, y)\), label one graph that most closely matches the formula:  (2 points each)

(A) \(f(x, y) = (-y)^2e^{-x^2-y^2}\)  (B) \(f(x, y) = e^x \cos(y)\)  (C) \(f(x, y) = \cos(x - y)\)
5. Let \( \mathbf{a} = (1, 1, 2), \mathbf{b} = (0, 3, 1) \). Check the boxes next to all (and only) the correct completions of the sentence. Mark all correct answer(s)! There may be more than one. (2 points)

“The quantity \(|\mathbf{a} \times \mathbf{b}|\)...

- ...is the volume of the parallelepiped spanned by \( \mathbf{a}, \mathbf{b}, \) and \( \mathbf{a} \times \mathbf{b} \)."
- ...is the area of the parallelogram spanned by \( \mathbf{a} \) and \( \mathbf{b} \)."
- ...is the area of the triangle with two sides \( \mathbf{a} \) and \( \mathbf{b} \)."
- ...is equal to \(|\mathbf{a}||\mathbf{b}|\cos(\theta)\)."
- ...is equal to \(|\mathbf{a}||\mathbf{b}|\sin(\theta)\)."
- ...is equal to \(|\mathbf{a} \cdot \mathbf{b}|\)."

6. Mark exactly one box corresponding to the correct ending to the sentence. (2 points)

“The limit \( \lim_{(x,y) \to (0,0)} \frac{x^3 y}{x^4 + y^4} \) fails to exist because...

- ...the numerator and denominator are both zero at (0,0)."
- ...the partial derivatives of \( \frac{x^3 y}{x^4 + y^4} \) at (0,0) do not exist." 
- ...the limits as one approaches (0,0) along the lines \( x = 0 \) and \( y = 0 \) are different." 
- ...the limits as one approaches (0,0) along the lines \( x = 0 \) and \( y = x \) are different." 
- ...the limits as one approaches (0,0) along the paths \( y = x^2 \) and \( x = 0 \) are different." 

7. Consider the function

\[
 f(x, y) = \begin{cases} 
 x^2 + y^2 & \text{for } x < 0 \\
 x & \text{for } x \geq 0 \text{ and } (x, y) \neq (0,0) \\
 1 & \text{for } (x, y) = (0,0) 
\end{cases}
\]

Check the box for the true statement (and check no other boxes). (2 points)

- \( f \) is continuous at (0,0).
- \( \lim_{(x,y) \to (0,0)} f(x, y) \) does not exist so \( f \) is discontinuous at (0,0).
- \( \lim_{(x,y) \to (0,0)} f(x, y) \) exists, but it is not equal to \( f(0,0) \), so \( f \) is discontinuous at (0,0).
8. Find a normal vector \( \mathbf{n} \) to the plane containing the points \((1,0,0), (0,2,0)\) and \((0,0,3)\).  \(3\) points

\[ \mathbf{n} = \langle \quad , \quad , \quad \rangle \]

9. Let \( f(x,y) \) be a function with values and derivatives in the table. Use linear approximation to estimate \( f(2.1,3.9) \).  \(3\) points

<table>
<thead>
<tr>
<th>( (x,y) )</th>
<th>( f(x,y) )</th>
<th>( \frac{\partial f}{\partial x}(x,y) )</th>
<th>( \frac{\partial f}{\partial y}(x,y) )</th>
<th>( \frac{\partial^2 f}{\partial x^2}(x,y) )</th>
<th>( \frac{\partial^2 f}{\partial y^2}(x,y) )</th>
<th>( \frac{\partial^2 f}{\partial x \partial y}(x,y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1,3))</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>-7</td>
<td>-2</td>
</tr>
<tr>
<td>((2,1))</td>
<td>2</td>
<td>-1</td>
<td>-3</td>
<td>-1</td>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>((2,4))</td>
<td>7</td>
<td>1</td>
<td>3</td>
<td>-1</td>
<td>-9</td>
<td>-5</td>
</tr>
<tr>
<td>((3,6))</td>
<td>1</td>
<td>-3</td>
<td>-5</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
</tr>
</tbody>
</table>

\( f(2.1,3.9) \approx \)

10. For which value of \( c \) is the volume of the parallelepiped determined by the vectors \( \langle 0,1,1 \rangle, \langle 2,3,5 \rangle, \) and \( \langle 1,1,c \rangle \) equal to zero?  \(3\) points

\[ c = \]

11. A plane \( P \) has equation \( 2x + 3y - z = 5 \). A line \( L \) is parameterized by \( \mathbf{r}(t) = \langle 1,1,0 \rangle + t\langle 2, a-1, a-3 \rangle \) for some number \( a \). Find the value of \( a \) for which the line \( L \) is contained in the plane \( P \).  \(2\) points

\[ a = \]
12. For functions $f: \mathbb{R}^2 \to \mathbb{R}$ and $r(t) = \langle x(t), y(t) \rangle$, let $F(t) = f(r(t)) = f(x(t), y(t))$.

(a) Write the Chain Rule formula \((2 \text{ points})\):

$$F'(t) = \ldots$$

(b) Suppose $f(x, y) = x^3 + 2xy + y^2 + x + y$.

(i) Compute $\frac{\partial f}{\partial x}(1, 0)$ and $\frac{\partial f}{\partial y}(1, 0)$ \((3 \text{ points})\):

$$\frac{\partial f}{\partial x}(1, 0) = \ldots$$

$$\frac{\partial f}{\partial y}(1, 0) = \ldots$$

(ii) Give the equation for the tangent plane to $f(x, y)$ at the point $(1, 0, 2)$. \((2 \text{ points})\)

$$\ldots x + \ldots y + \ldots z = \ldots$$

(iii) Let $r(t) = \langle x(t), y(t) \rangle$ be the position of a particle at time $t$. Let $F(t) = f(r(t))$. Suppose $r(0) = \langle 1, 0 \rangle$. Give any possible nonzero velocity vector $r'(0) = \langle x'(0), y'(0) \rangle$ of the particle at time $t = 0$ which would imply that $F'(0) = 0$. \textbf{You must justify your answer.} \((2 \text{ points})\)

$$r'(0) = \langle \ldots , \ldots \rangle$$

13. Compute the distance from the point $(1, 3, 1)$ to the plane whose equation is $2x + y - z = 16$. \((3 \text{ points})\)

$$\text{distance} = \ldots$$
14. **Extra Credit Problem.** Let \( f(x, y) = \frac{x\sin^2(x)}{x^2 + y^2} \).

(a) Find a \( \delta > 0 \) such that if \( 0 < |\langle x, y \rangle| < \delta \) then \( |f(x, y)| < \frac{1}{10} \). Justify your answer. (1 point)

(b) Find an expression for \( \delta > 0 \) in terms of \( \epsilon \) so that for every \( \epsilon > 0 \), if \( 0 < |\langle x, y \rangle| < \delta \) then \( |f(x, y)| < \epsilon \). Justify your answer. (1 point)

Scratch work may go below and on the back of this sheet.
Scratch work may go below.