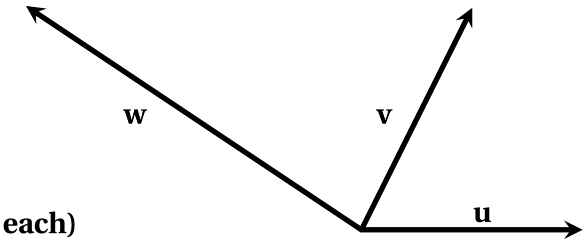
1. Consider the vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ in $\mathbb{R}^2$ shown at right.



For each of the following, circle the best answer **(2 points each)**

(a) $\mathbf{w} =$ | $\mathbf{u}+2\mathbf{v}$ | $2\mathbf{u}+\mathbf{v}$ | $-2\mathbf{u}+\mathbf{v}$ | $-\mathbf{u}+2\mathbf{v}$ | $\mathbf{u}-2\mathbf{v}$ | $2\mathbf{u}-\mathbf{v}$

(b) $\text{proj}_{\mathbf{u}}\mathbf{v} =$ | $2\mathbf{u}$ | $\frac{1}{2}\mathbf{u}$ | $2\mathbf{v}$ | $\frac{1}{2}\mathbf{v}$ | $2\mathbf{w}$ | $\mathbf{w}+\mathbf{v}$ | $-2\mathbf{v}$

(c) $\mathbf{u}\cdot\mathbf{w}$ | $>$ / $<$ / $=$ | 0.

2. Suppose $f(x, y)$ has the contour plot below right, with points labelled. Circle the best answer to each of the following questions: **(1 point each)**

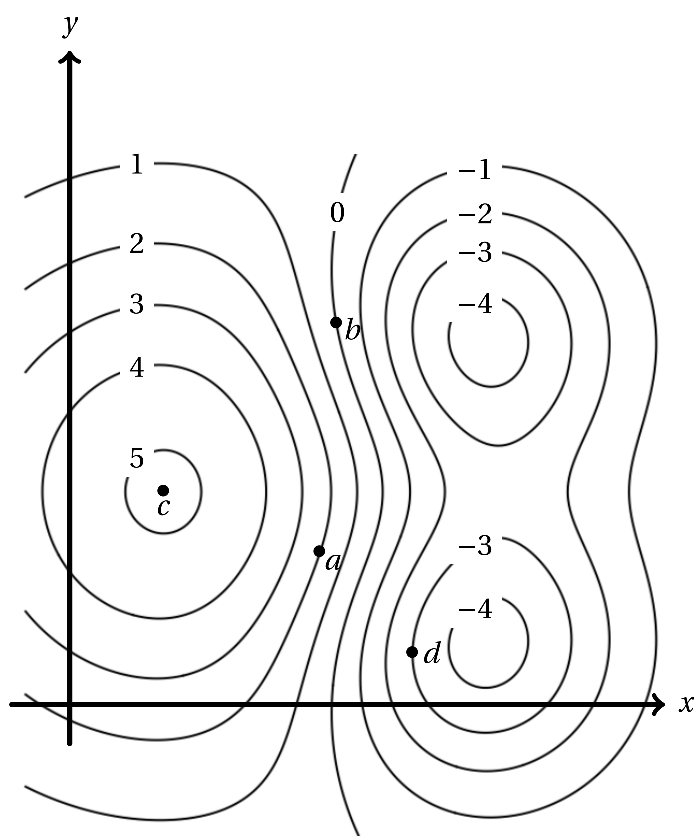
(a) $f(c)$ is: **positive** **negative** **0**

(b) $\dfrac{\partial f}{\partial x}(b)$ is: **positive** **negative** **0**

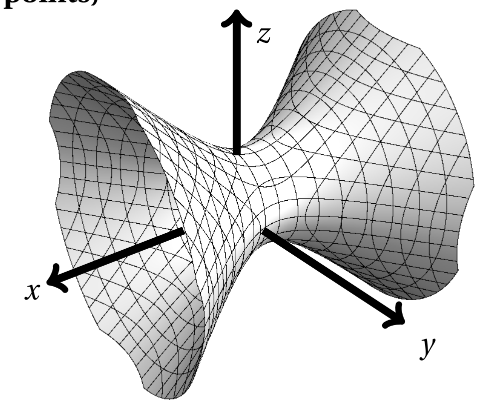
(c) $\dfrac{\partial^2 f}{\partial y^2}(d)$ is: **positive** **negative** **0**

(d) Circle one:

$\dfrac{\partial f}{\partial x}(a) > \dfrac{\partial f}{\partial x}(c)$ $\qquad$ $\dfrac{\partial f}{\partial x}(a) < \dfrac{\partial f}{\partial x}(c)$



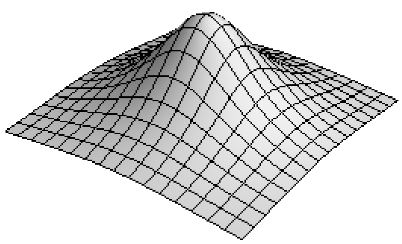
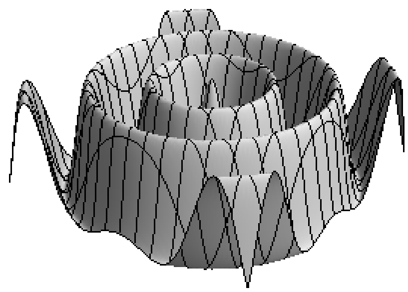
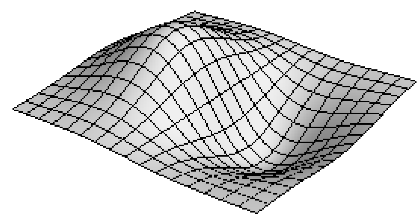
3. Circle the equation for the quadric surface shown at right. **(2 points)**

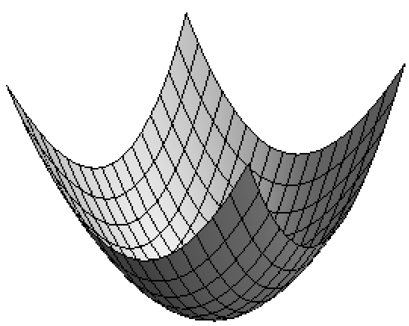
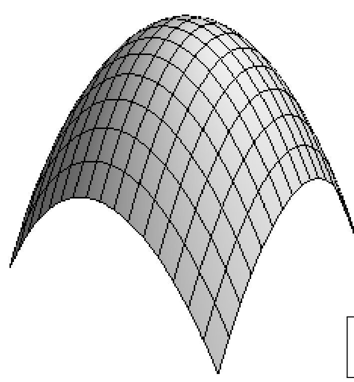
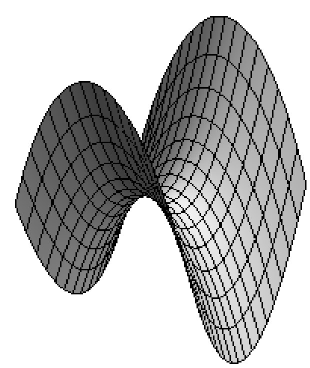


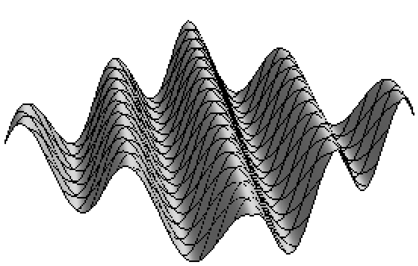
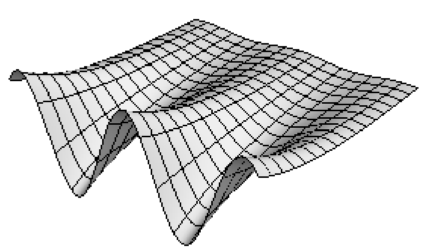
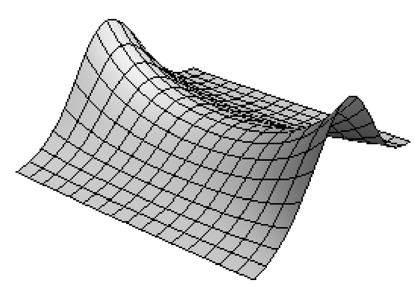
1. $x^2 - y^2 + z^2 = 1$
2. $-x^2 + y^2 + z^2 = 1$
3. $x^2 + y^2 - z^2 = -1$
4. $x - y^2 + z = 1$
5. $x^2 - y^2 - z^2 = 1$

4. For each function $f(x, y)$, label one graph that most closely matches the formula: **(2 points each)**

(A) $f(x, y) = (-y)2^{(-x^2-y^2)}$ (B) $f(x, y) = e^x \cos(y)$ (C) $f(x, y) = \cos(x - y)$

5. Let $\mathbf{a} = \langle 1,1,2\rangle$, $\mathbf{b} = \langle 0,3,1\rangle$. Check the boxes next to *all* (and only) the correct completions of the sentence. **Mark all correct answer(s)! There may be more than one. (2 points)**

"The quantity $|\mathbf{a}\times\mathbf{b}|$...

- [ ] ... is the volume of the parallelepiped spanned by $\mathbf{a}, \mathbf{b}$, and $\mathbf{a}\times\mathbf{b}$."
- [ ] ...is the area of the parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$."
- [ ] ...is the area of the triangle with two sides $\mathbf{a}$ and $\mathbf{b}$."
- [ ] ...is equal to $|\mathbf{a}||\mathbf{b}|\cos(\theta)$."
- [ ] ...is equal to $|\mathbf{a}||\mathbf{b}|\sin(\theta)$."
- [ ] ...is equal to $|\mathbf{a}\cdot\mathbf{b}|$."

6. Mark exactly one box corresponding to the correct ending to the sentence. **(2 points)**

"The limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^4+y^4}$ fails to exist because...

- [ ] ...the numerator and denominator are both zero at $(0,0)$."
- [ ] ...the partial derivatives of $\dfrac{x^3 y}{x^4+y^4}$ at $(0,0)$ do not exist."
- [ ] ...the limits as one approaches $(0,0)$ along the lines $x=0$ and $y=0$ are different."
- [ ] ...the limits as one approaches $(0,0)$ along the lines $x=0$ and $y=x$ are different."
- [ ] ...the limits as one approaches $(0,0)$ along the paths $y=x^2$ and $x=0$ are different."

7. Consider the function

$$f(x,y) = \begin{cases} x^2+y^2 & \text{for } x<0 \\ x & \text{for } x\ge 0 \text{ and } (x,y)\neq(0,0) \\ 1 & \text{for } (x,y)=(0,0) \end{cases}$$

Check the box for the true statement (and check no other boxes). **(2 points)**

- [ ] f is continuous at $(0,0)$.
- [ ] $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist so f is discontinuous at $(0,0)$.
- [ ] $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ exists, but it is not equal to $f(0,0)$, so f is discontinuous at $(0,0)$.

8. Find a normal vector $\mathbf{n}$ to the plane containing the points $(1,0,0)$, $(0,2,0)$ and $(0,0,3)$. **(3 points)**

$\mathbf{n} = \langle \quad , \quad , \quad \rangle$

9. Let $f(x, y)$ be a function with values and derivatives in the table. Use linear approximation to estimate $f(2.1, 3.9)$. **(3 points)**

(x, y)	$f(x, y)$	$\frac{\partial f}{\partial x}(x, y)$	$\frac{\partial f}{\partial y}(x, y)$	$\frac{\partial^2 f}{\partial x^2}(x, y)$	$\frac{\partial^2 f}{\partial y^2}(x, y)$	$\frac{\partial^2 f}{\partial x \partial y}(x, y)$
$(-1,3)$	0	4	4	2	-7	-2
$(2,1)$	2	-1	-3	-1	-1	-2
$(2,4)$	7	1	3	-1	-9	-5
$(3,6)$	1	-3	-5	0	-1	-2

$f(2.1, 3.9) \sim$

10. For which value of c is the volume of the parallelepiped determined by the vectors $\langle 0,1,1\rangle$, $\langle 2,3,5\rangle$, and $\langle 1,1,c\rangle$ equal to zero? **(3 points)**

$c =$

11. A plane P has equation $2x+3y-z=5$. A line L is parameterized by $\mathbf{r}(t) = \langle 1,1,0\rangle + t\langle 2, a-1, a-3\rangle$ for some number a. Find the value of a for which the line L is contained in the plane P. **(2 points)**

$a =$

12. For functions $f\colon \mathbb{R}^2 \to \mathbb{R}$ and $\mathbf{r}(t) = \langle x(t), y(t)\rangle$, let $F(t) = f(\mathbf{r}(t)) = f(x(t), y(t))$.

(a) Write the Chain Rule formula **(2 points)** :

$F'(t) =$

(b) Suppose $f(x, y) = x^3 + 2xy + y^2 + x + y$.

(i) Compute $\frac{\partial f}{\partial x}(1,0)$ and $\frac{\partial f}{\partial y}(1,0)$ **(3 points)** :

$\frac{\partial f}{\partial x}(1,0) =$

$\frac{\partial f}{\partial y}(1,0) =$

(ii) Give the equation for the tangent plane to $f(x, y)$ at the point $(1,0,2)$. **(2 points)**

$\square\, x + \square\, y + \square\, z = \square$

(iii) Let $\mathbf{r}(t) = \langle x(t), y(t)\rangle$ be the position of a particle at time t. Let $F(t) = f(\mathbf{r}(t))$. Suppose $\mathbf{r}(0) = \langle 1,0\rangle$. Give any possible nonzero velocity vector $\mathbf{r}'(0) = \langle x'(0), y'(0)\rangle$ of the particle at time $t = 0$ which would imply that $F'(0) = 0$. **You must justify your answer.** **(2 points)**

$\mathbf{r}'(0) = \langle \quad , \quad \rangle$

13. Compute the distance from the point $(1,3,1)$ to the plane whose equation is $2x + y - z = 16$. **(3 points)**

distance =

14. Extra Credit Problem. Let $f(x,y) = \dfrac{x\sin^2(x)}{x^2+y^2}$.

(a) Find a $\delta > 0$ such that if $0 < |\langle x,y\rangle| < \delta$ then $|f(x,y)| < \dfrac{1}{10}$. Justify your answer. **(1 point)**

(b) Find an expression for $\delta > 0$ in terms of ϵ so that for *every* $\epsilon > 0$, if $0 < |\langle x,y\rangle| < \delta$ then $|f(x,y)| < \epsilon$. Justify your answer. **(1 point)**

Scratch work may go below and on the back of this sheet.

Scratch work may go below.