1. Suppose $f(x,y)$ has values and partial derivatives as in the table at right.

(a) Find all the critical points you can from the given data and classify them into local mins, local maxes, and saddles. **(3 points)**

(x,y)	f	f_x	f_y	f_{xx}	f_{xy}	f_{yy}
(1,0)	3	0	0	-2	-1	-3
(0,1)	2	0	1	1	0	2
(1,4)	5	0	0	-1	3	2
(2,0)	0	-1	0	1	0	-2
(2,2)	2	0	0	-1	1	-2

Local mins (if any) at:

Local maxes (if any) at:

Saddles (if any) at:

(b) Let $L(x,y)$ denote the linear approximation to $f(x,y)$ at the point $(2,0)$. Is $L(2.1,0)$ likely to be larger than, equal to, or smaller than $f(2.1,0)$? Circle your answer below. **(1 point)**

$L(2.1,0) > f(2.1,0)$ $\quad$ $L(2.1,0) = f(2.1,0)$ $\quad$ $L(2.1,0) < f(2.1,0)$

2. Let $g(x,y,z)$ be the function with table of values and derivatives below.

(x,y,z)	$g(x,y,z)$	g_x	g_y	g_z	g_{xx}	g_{xy}	g_{yy}	g_{zz}
(1,0,1)	2	2	1	1	0	1	0	1
(2,1,1)	1	2	4	-3	1	-2	3	1
(2,0,1)	1	-2	-4	3	0	2	8	9
(1,1,2)	1	4	-1	3	-3	2	-1	-2

(a) Find an equation of the tangent plane to the level surface $g(x,y,z) = 1$ at the point $(2,1,1)$. Show work justifying your answer. **(2 points)**

Equation: $\square\, x + \square\, y + \square\, z = \square$

(b) Find the directional derivative $D_{\mathbf{u}}g(1,1,2)$, where $\mathbf{u} = \frac{1}{\sqrt{14}}\langle 1,3,2\rangle$. Show work justifying your answer. **(2 points)**

$D_{\mathbf{u}}g(1,1,2) =$

3. Circle the correct response: The vector field

$$\mathbf{F} \;=\; xy\,\mathbf{i} + xy\,\mathbf{j} \;=\; \langle xy, xy \rangle$$

on $\mathbf{R}^2$ is: | conservative | not conservative |

Justify your answer with a calculation. **(3 points)**

4. Find a potential function f for the conservative vector field

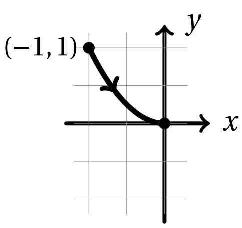
$$\mathbf{F} \;=\; (e^x\cos(y) + y)\,\mathbf{i} + (-e^x\sin(y) + x + 1)\,\mathbf{j} \;=\; \langle e^x\cos(y) + y, -e^x\sin(y) + x + 1 \rangle.$$

(3 points) No partial credit – you can check your answer!

$f(x, y) =$

5. Let C be the curve in $\mathbb{R}^2$ parameterized by $\mathbf{r}(t) = \langle -t^2, -t \rangle$ for $-1 \le t \le 0$.

(a) Mark the picture of C from among the choices below. **(1 point)**



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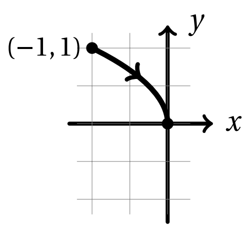

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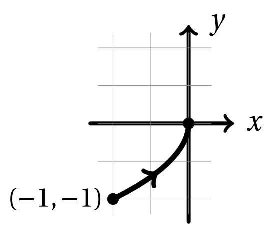

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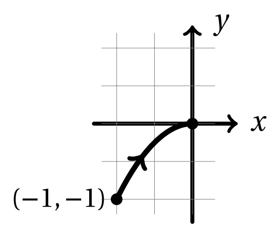

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(b) For the vector field $\mathbf{F} = \langle x+3, y \rangle$ directly calculate $\int_C \mathbf{F}\cdot d\mathbf{r}$ using the given parameterization. **(4 points)**

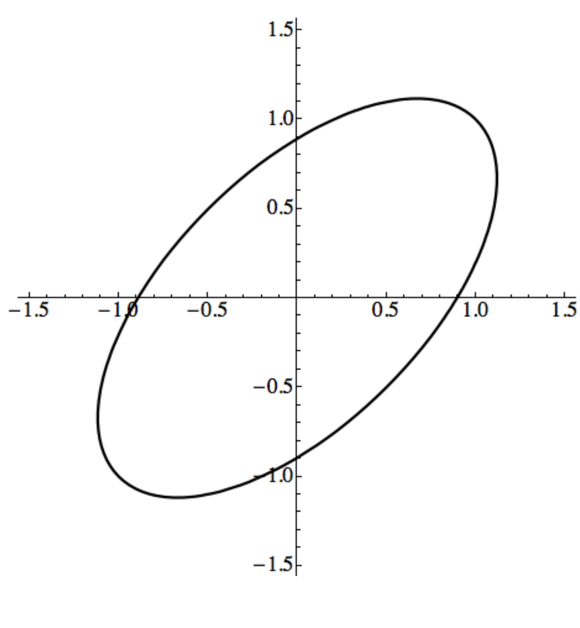
$\int_C \mathbf{F}\cdot d\mathbf{r} =$

6. Let $f(x,y) = 2xy$.

(a) Use the method of Lagrange multipliers to find the maximum value M and minimum value m of the function $f(x,y)$, subject to the constraint $5x^2 - 6xy + 5y^2 = 4$. **(5 points)**

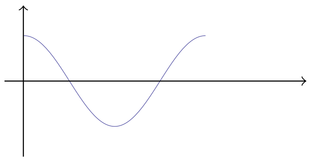
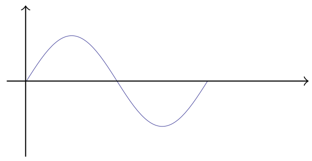
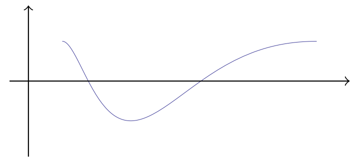
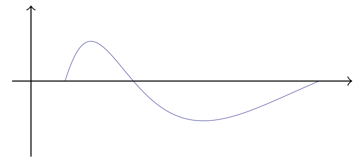
$M =$	$m =$

(b) The curve $5x^2 - 6xy + 5y^2 = 4$ is pictured below. On the same graph, sketch the level set $f(x,y) = M$. **(2 points)**



7. Consider the oriented curve C given by the parameterization $\mathbf{r}(t) = (e^t, \cos(\pi t))$ for $0 \le t \le 2$.
Circle the correct answer for each of the following.

(a) Which figure most accurately illustrates the curve C? **(2 points)**

(b) If $\mathbf{F}(x,y) = (2 + e^{y\sin(x)}, 0)$, then the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is | negative | zero | positive | **(2 points)**

(c) Which of the following calculates the length of C? **(2 points)**

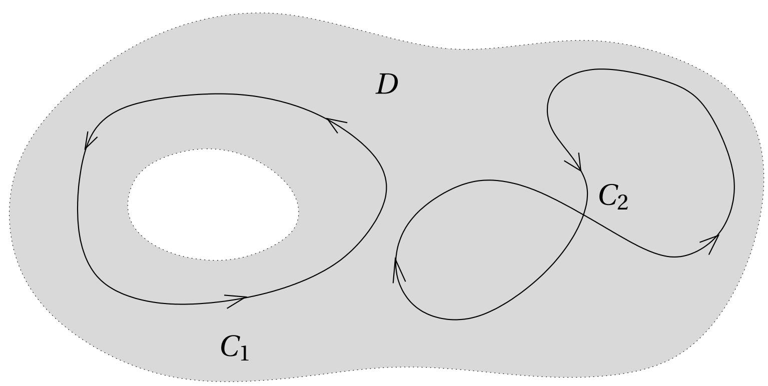
$\int_0^2 \sqrt{e^{2t} + \pi^2 \cos^2(\pi t)}\, dt$ $\quad \int_0^2 \sqrt{e^{t} + \pi^2 \sin^2(\pi t)}\, dt$ $\quad \int_0^2 \sqrt{e^{2t} + \cos^2(\pi t)}\, dt$ $\quad \int_0^2 \sqrt{e^{2t} + \pi^2 \sin^2(\pi t)}\, dt$

8. Let D be the open subset of the plane containing the two closed curves C_1 and C_2 shown below right.

Suppose $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j}$ and $G = R\,\mathbf{i} + S\,\mathbf{j}$ are vector fields on D satisfying

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \qquad \frac{\partial R}{\partial y} \neq \frac{\partial S}{\partial x}, \qquad \int_{C_1} \mathbf{F} \cdot d\mathbf{r} = 13, \qquad \int_{C_1} \mathbf{G} \cdot d\mathbf{r} = 0$$

For each of the following statements, decide whether or not it is necessarily true or false based on the information above. **(1 point each)**



(a) D is simply connected. | True | False |

(b) **F** must be conservative. | True | False |

(c) **G** must be conservative. | True | False |

(d) If $-C_1$ is the same curve as C_1 but parameterized in the opposite direction, then $\int_{-C_1} \mathbf{F} \cdot d\mathbf{r}$ must be nonzero. | True | False |

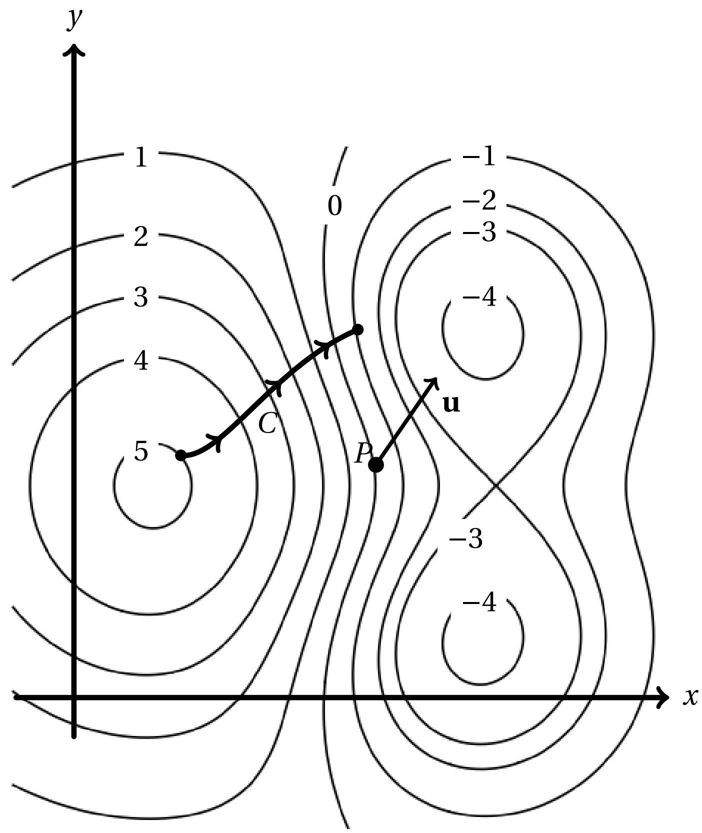
(e) There must be some curve C_3 so that $\int_{C_3} \mathbf{G} \cdot d\mathbf{r} \neq 0$. | True | False |

(f) (**Extra credit**) $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ must be zero. | True | False |

9. Find a parameterization $\mathbf{r}(t)$ of the curve of intersection of the cylinder $\frac{y^2}{4}+z^2=1$ and the plane $x-y+z=1$; specify the range of the parameter t. **(4 points)**

$\mathbf{r}(t) = \langle \qquad , \qquad , \qquad \rangle$ for $\square \le t \le \square$

10. Consider the function $f(x,y)$ whose contour diagram on the open set $D = \{(x,y) \mid 0 < x < 4,\ 0 < y < 4\}$ is shown below right. For each part, circle the best answer. **(1 point each)**



(a) The integral $\int_C f\, ds$ is:

negative	zero	positive

(b) The integral $\int_C \nabla f \cdot d\mathbf{r}$ is:

−8	−6	−4	−2	0	2	4	6	8

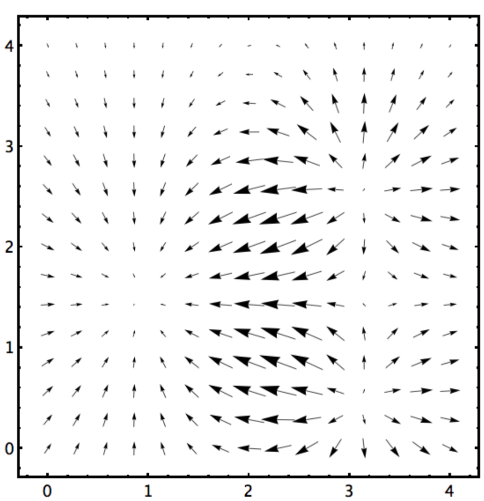
(c) The value of $D_{\mathbf{u}} f(P)$ is:

negative	zero	positive

(d) The number of critical points of f in D is:

0	1	2	3	4	5	6

(e) Mark the plot below of the gradient vector field ∇f.



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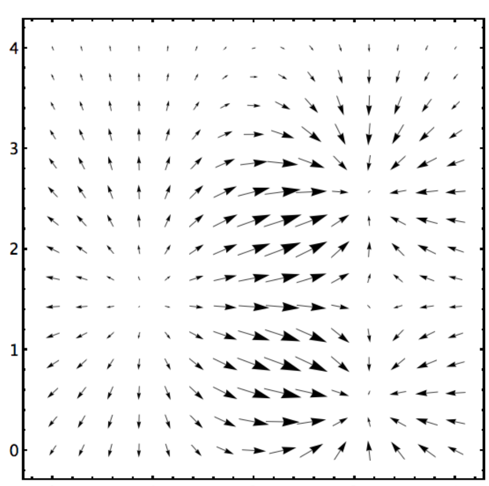

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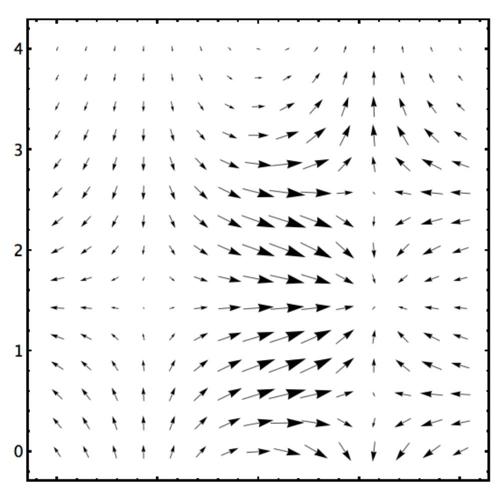

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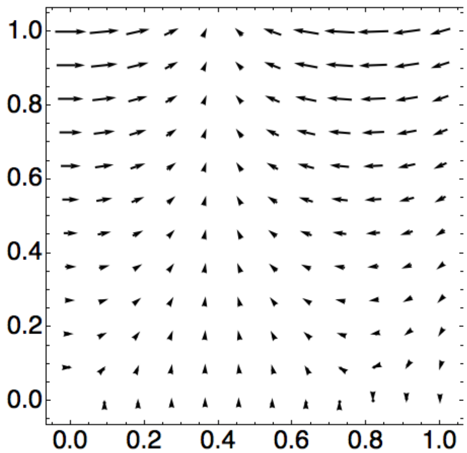
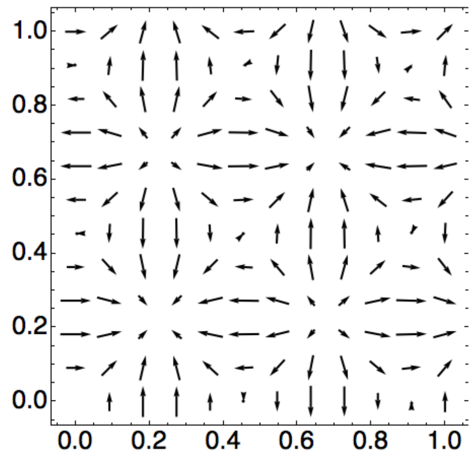
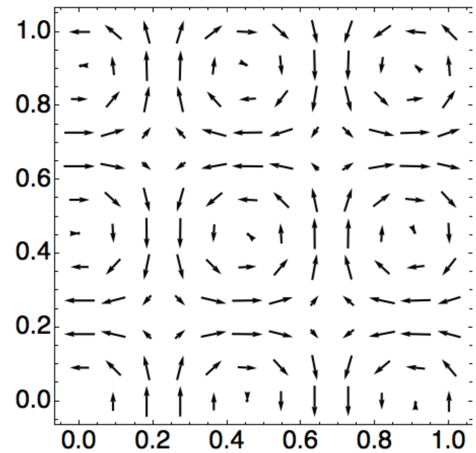
11. Consider the three vector fields **E**, **F**, and **G** on $\mathbb{R}^2$ shown below.



E



F



G

(i) One of these vector fields is $\cos(7x)\sin(7y)\mathbf{i} + \sin(7x)\cos(7y)\mathbf{j}$. Circle its name: **(1 point)**

E **F** **G**

(ii) Exactly one of these vector fields is **not** conservative. Circle it here: **E** **F** **G** **(1 point)**

Scratch work may go here