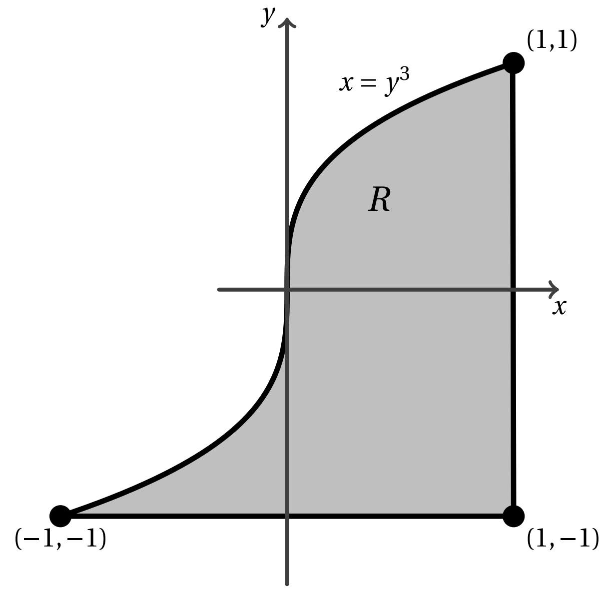
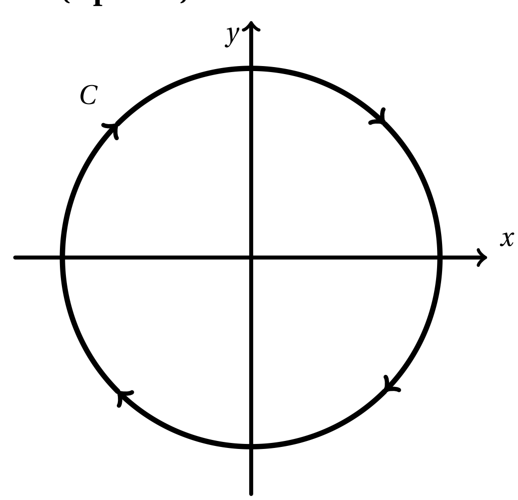
1. Let R denote the shaded region pictured below right. Compute $\iint_R 14x\,dA$. **(4 points)**



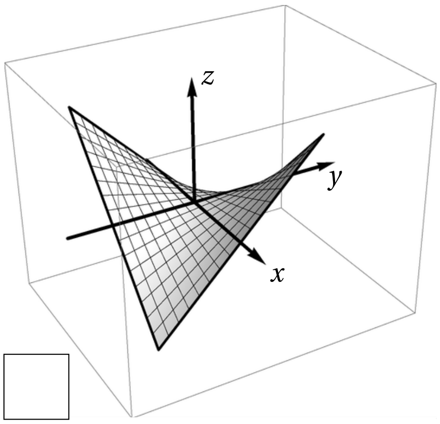
$\iint_R 14x\,dA =$

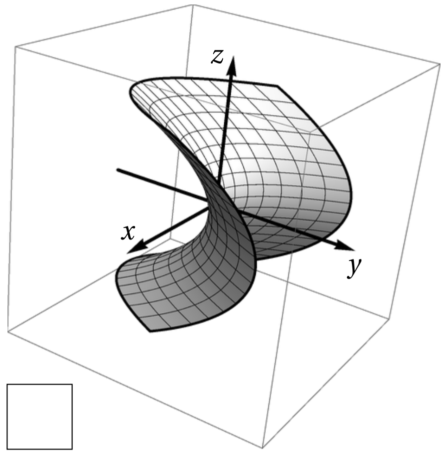
2. Let C be the circle of radius 1 centered at the origin and oriented as shown below right. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y) = \left(e^x y^2 - x^2 y, 2e^x y + x y^2\right)$ by any valid method. **(3 points)**

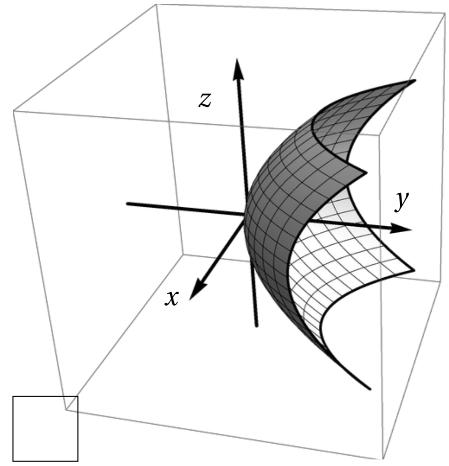


$\int_C \mathbf{F}\cdot d\mathbf{r} =$

3. Let S be the surface parameterized by $\mathbf{r}(u,v) = \langle u, u^2 - v^2, v \rangle$ for $-1 \le u \le 1$ and $-1 \le v \le 1$. Mark the picture of S below. **(2 points)**







4. Consider the surface S parameterized by $\mathbf{r}(u,v) = (uv, v, u^2)$ with $-1 \le u \le 1$ and $0 \le v \le 1$.

(a) **(2 points)** Choose the integrand that correctly describes the integral over S of the function y, and circle your answer. Make sure to show work that justifies your choice.

$$\iint_S y\, dS = \int_0^1 \int_{-1}^1 \quad [\ \text{choice}\ ] \quad du\, dv$$

$v\sqrt{4u^2 + 4u^4 + v^2}$	$v\sqrt{u^2v^2 + v^2 + u^4}$
$u\sqrt{4u^2 + 4u^4 + v^2}$	$v\sqrt{4u^2 + 4u^4 + v}$

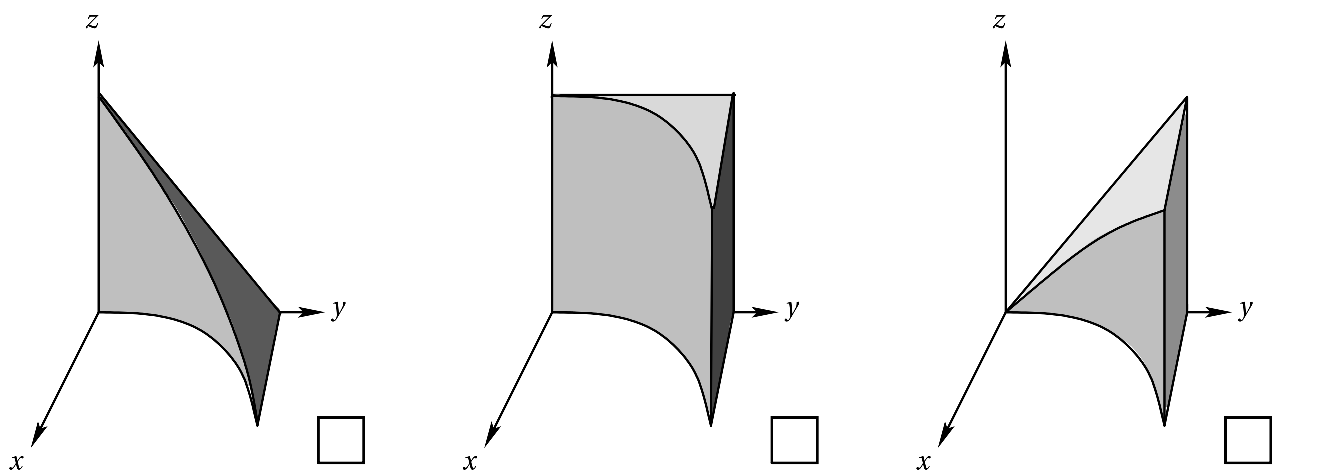
(b) **(2 points)** Circle the true statement:

$\iint_S y\, dS < \iint_S y^2\, dS$	$\iint_S y\, dS = \iint_S y^2\, dS$	$\iint_S y\, dS > \iint_S y^2\, dS$

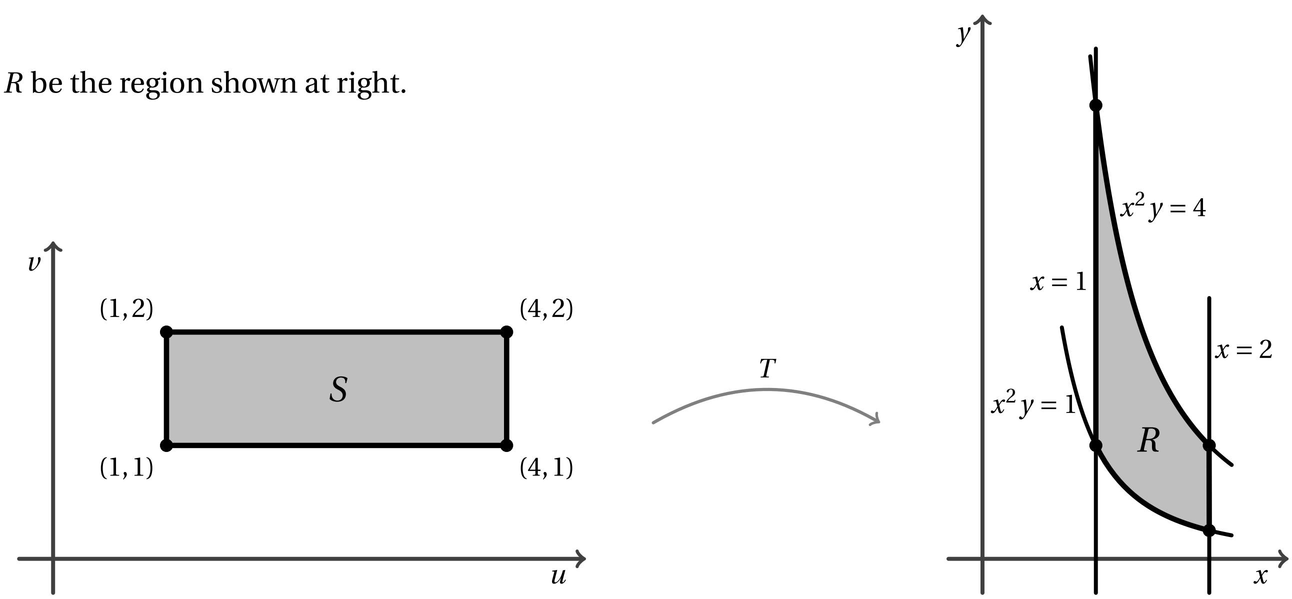
5. (a) **(2 points)** Calculate $\int_0^1 \int_0^{y^2} \int_0^y 2\,dz\,dx\,dy$.

$\int_0^1 \int_0^{y^2} \int_0^y 2\,dz\,dx\,dy =$

(b) **(2 points)** Decide which region is being integrated over in part (a), and check the corresponding box.



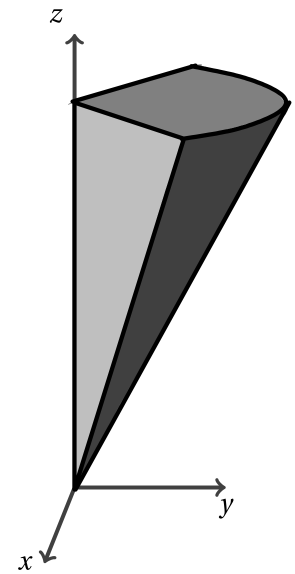
6. Let R be the region shown at right.



Find a transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ taking $S = [1,4] \times [1,2]$ to R. Justify your answer. **(3 points)**

$T(u,v) = \Big\langle \qquad\qquad , \qquad\qquad \Big\rangle$

7. Let R be the solid region in $\mathbb{R}^3$ shown at the right. Specifically, R is bounded by the cone $z^2 = 4x^2 + 4y^2$, the planes $x + y = 0$, $x - y = 0$, $z = 2$, and has $y, z \geq 0$. The volume of R is calculated in cylindrical coordinates by the integral



$$\int_A^B \int_C^D \int_E^F G\,dz\,dr\,d\theta.$$

Determine the quantities A, B, C, D, E, F, G and circle the correct answers.

(a) **(1 point)**

$A =$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$B =$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π

(b) **(1 point)**

$C =$	0	1	2	3	4
$D =$	0	1	2	3	4

(c) **(1 point)** $E =$ | 0 | $2r$ | $4r^2$ | 2 |

(d) **(1 point)** $F =$ | 0 | $2r$ | $4r^2$ | 2 |

(e) **(1 point)** $G =$ | 0 | 1 | r | r^2 | $r\sin(\theta)$ | $r^2\sin(\theta)$ | $r^2\cos(\theta)$ |

8. Use spherical coordinates to SET UP, but not calculate, $\iiint_B z\,dV$, where B is the part of the unit ball in $\mathbb{R}^3$, $x^2 + y^2 + z^2 \leq 1$, for which $x \leq 0$. **(4 points)**

$$\iiint_B z\,dV = \int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} \Big(\qquad\qquad\qquad \Big)\, d\rho\, d\phi\, d\theta$$

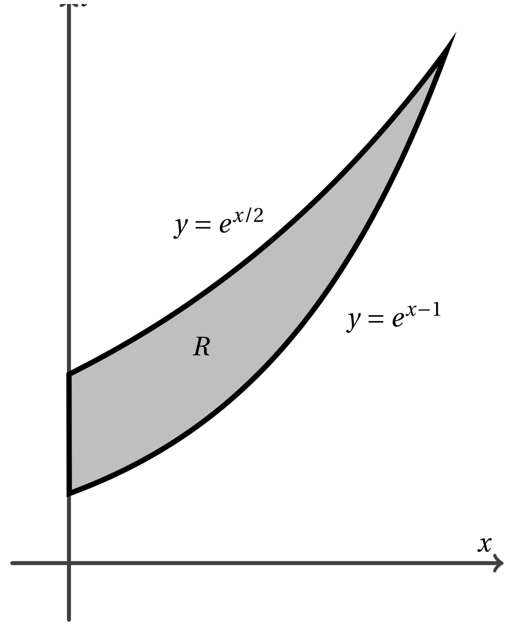
9. Let R be the region shown at right and consider the transformation

$$T(u,v) = \left(u+v, e^u\right).$$

Using the transformation to change coordinates, determine the quantities B, C, D, E, F in the expression

$$\iint_R \frac{x}{y}\, dA = \int_B^C \int_D^E \quad F \quad du\, dv$$

and **circle the correct answer**.



(a) **(1 point)** $B =$ | 0 1 2 u v $-u$ $-v$ |

(b) **(1 point)** $C =$ | 0 1 2 u v $-u$ $-v$ |

(c) **(1 point)** $D =$ | 0 1 2 u v $-u$ $-v$ |

(d) **(1 point)** $E =$ | 0 1 2 u v $-u$ $-v$ |

(e) **(2 points)** $F =$ | $(u+v)e^u$ $(u+v)e^{-u}$ $u+v$ $-(u+v)e^u$ $-(u+v)e^{-u}$ |

MORE PROBLEMS ON THE NEXT PAGE!

10. For each surface S in parts (a) and (b), give a parameterization $\mathbf{r}\colon D \to S$. Be sure to explicitly specify the domain D and call your parameters u and v.

(a) The part of the surface in $\mathbb{R}^3$ defined by $y = z^2 - x^2$ with $-1 \le x \le 1$ and $0 \le z \le 2$. **(3 points)**

$D = \{ \quad \le u \le \quad$ and $\quad \le v \le \quad \}$

$\mathbf{r}(u, v) = \langle \quad , \quad , \quad \rangle$

(b) The portion of the sphere $x^2 + y^2 + z^2 = 1$ in $\mathbb{R}^3$ where $z \le 0$. **(3 points)**

$D = \{ \quad \le u \le \quad$ and $\quad \le v \le \quad \}$

$\mathbf{r}(u, v) = \langle \quad , \quad , \quad \rangle$

(c) The portion of the surface in $\mathbb{R}^3$ defined by $z^2 - y^2 + 1 = 0$ with $y \ge 0$ that lies inside the cylinder $x^2 + z^2 = 1$. **(3 points)**

$D = \{ \quad \le u \le \quad$ and $\quad \le v \le \quad \}$

$\mathbf{r}(u, v) = \langle \quad , \quad , \quad \rangle$