AERODYNAMIC PROPERTIES OF INDIVIDUAL FERTILIZER PARTICLES

T. E. Grift, J. T. Walker, J. W. Hofstee

ABSTRACT. *Predictability of granular fertilizer spreading patterns is of interest from the environmental as well as the economic point of view. To ensure a constant level of uniformity of spreading patterns in the field, the Dutch government has announced their intention to require periodic testing of spreader equipment. Testing of fertilizer spreaders is traditionally carried out in large halls where spread patterns are derived from measuring fertilizer mass in collecting bins. Hofstee (1994) has developed an alternative system which measures three-dimensional velocity vectors within a cylindrical sampling zone behind the spreader. It also simultaneously estimates individual particle diameters. These measured quantities serve as initial conditions in a trajectory model that predicts landing spots for individual particles. After a test run the complete set of landing spots represents a spread pattern. The trajectory model uses prediction equations based on the aerodynamic behavior of perfectly spherical particles. However, since fertilizer particles are in general not spherical, a method to compensate for this has been developed. This method uses the ratio between measured and modeled fall times, and is expressed in a parameter, the diameter coefficient. Once this parameter is assessed for a specific material, it can be used as a correction factor in the trajectory model. In this research a fall test is used as a robust and simple method for collecting data about the fall time of individual fertilizer particles, falling from a constant height. The materials used in this research were Calcium Ammonium Nitrate (CAN 27 N), Nitrate Phosphorous Potassium (NPK 12-10-18) and Potassium 60. They were chosen for their wide-spread use and different shape characteristics. The diameter range of particles used in the research was 1 to 4.75 mm.*
Keywords. *Fertilizer, Granular, Spreading, Aerodynamic, Fall test.*

The Dutch government has announced the intention to require a periodic inspection of fertilizer spreaders used by farmers and/or contractors regarding their spreading performance (Heestermans, 1993). Proper testing of fertilizer spreaders requires large indoor test facilities to exclude environmental influences such as wind and rain. These test facilities are expensive, therefore an alternative test arrangement has been developed by Hofstee (1994) which allows tests to be performed in a small space. This arrangement no longer requires actual catching of the fertilizer material in bins, but is based on measurement of the velocity and diameter of fertilizer particles as they pass through an orifice in a metal plate. This plate is mounted perpendicular to the expected particle trajectories and is moved around the spreader in a cylindrical manner. These measurements serve as inputs for a trajectory model which predicts the landing position of individual particles. These predictions can be accumulated to yield a spread pattern. Thus, an estimate of fertilizer spreader performance could be made in a relatively small area under controlled conditions. However, the trajectory model requires prior knowledge of aerodynamic characteristics of fertilizer particles. Some of these particle properties are neither well known, nor easy to determine.

Basic physical formulas describe aerodynamic behavior of homogenous objects with distinct geometric shapes (Mohsenin, 1980). The forces acting on these objects are proportional to particle characteristics (mass, frontal area, and drag coefficient) as well as air density and instantaneous velocity. The drag coefficient is an empirical number, the value of which depends upon the particle shape and Reynolds number. Aerodynamic forces may also be influenced by the momentary orientation or spinning of the particles. The shape of fertilizer particles like Calcium Ammonium Nitrate (CAN) can be regarded as near-spherical with a certain surface roughness. Other materials like Potassium are more plate-like with distinct edges. Exact prediction of aerodynamic properties of fertilizer may require investigation of individual particle properties and shape.

The aerodynamic performance of individual particles may be investigated by wind tunnel, elutriator, or fall test. Forces acting on individual particles are extremely small and vary over a large range due to the squared relationship with velocity, making accurate measurements in a wind tunnel quite difficult. Several investigators have used a vertical wind tunnel called an elutriator (Law and Collier, 1973) in which a particle is supported in air moving at 'terminal' velocity. However, unlike a wind tunnel, the elutriator

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provides information for only a single velocity. The fall test (Bilanski et al., 1962; Keck and Goss, 1965) provides a measure of aerodynamic performance through the interval of time required for a particle to fall a known distance. In the fall test, particles accelerate from velocity zero to (asymptotically) terminal velocity and the aerodynamic performance is therefore an "average" for the conditions during the fall test. In ground-based fertilizer spreaders, particles are launched at a velocity much higher than terminal velocity (up to 70 ms^{-1}) and decelerate towards it, which means that the results of the fall tests have to be extrapolated to the real life situation. Even with this limitation, the fall test is a valuable tool since it can provide rapid measurements for a large number of particles.

The purpose of this study was to develop and experimentally evaluate a theory for the aerodynamic performance of spherical particles and to determine the variability of the aerodynamic characteristics of fertilizer particles. The overall aim of the research was to quantify the effects of the variation of particle aerodynamic properties within bulk fertilizer materials on deposition. This knowledge is essential for development of trajectory models of sufficient accuracy to perform tests on fertilizer spreaders without actually catching the material in spaced bins.

THEORY

In the theoretical approach, particles are regarded as homogenous spheres with a specific density and a distinct diameter. Buoyancy effects were neglected in the model because of the large difference in density between fertilizer and air. Under this assumption, three forces are acting on a particle when it falls in still air, a gravitational force, an aerodynamic force and a resulting inertia force. These forces will at all times compensate each other as shown in figure 1. The gravitational force is equal to the mass times the gravitational acceleration, symbolized F_g. The aerodynamic drag force is equal to the drag coefficient times the frontal area of a particle, times the dynamic pressure term or $F_D = C_D A[(1/2)\rho_{air}\dot{y}^2]$. The inertia force acting on a particle equals the mass of the particle times the acceleration it is subjected to, or $F_I = m\ddot{y}$. The force balance then yields:

$$F_I + F_D = F_g \qquad (1)$$

Substitution of the formulas given for the components leads to:

$$m\ddot{y} + C_D A \frac{1}{2} \rho_{air} \dot{y}^2 = mg \qquad (2)$$

This equation can be simplified by substitution of:

$$K = \left(\frac{1}{2}\frac{C_D A \rho_{air}}{m}\right) \qquad (3)$$

leading to,

$$\ddot{y} + K\dot{y}^2 = g. \qquad (4)$$

Also from this equation, terminal velocity follows, since in a stationary situation the acceleration term will become zero, leading to:

$$K\dot{y}^2 = g \text{ or } \dot{y} = \sqrt{(g/K)}$$

m = mass (kg)
y = vertical position (m)
C_D = drag coefficient, 1
A = particle frontal area (m^2)
ρ_{air} = air density (at 20°C) (kgm^{-3})
g = local gravitational acceleration (ms^{-2})
ρ_p = particle density (kgm^{-3})
r_p = particle radius (m)

Equation 4 is a second-order ordinary differential equation which is non-linear due to the square in the $K\dot{y}^2$ term. The general solution of equation 4 was found using MatLab (1995).

$$y(t) = \frac{1}{K}\{\ln[\cosh(\sqrt{gK}(t - C_1))]\} + C_2 \qquad (5)$$

With $y(0) = y_0$ and $\dot{y}(0) = v_0$, the constants C_1 and C_2 were found to be:

$$C_1 = \frac{-1}{\sqrt{gK}}\left[\text{arctanh}\left(\sqrt{\frac{K}{g}}\, v_0\right)\right] \qquad (6)$$

$$C_2 = y_0 - \frac{1}{K}\left\{\ln\left[\cosh\left(\text{arctanh}\left(\sqrt{\frac{K}{g}}\, v_0\right)\right)\right]\right\} \qquad (7)$$

In this simple model, the drag coefficient was regarded as a constant. In reality, it depends on the Reynolds number (Re). Therefore, the total fall time was computed by application of the simple model only for a small distance interval. The length of the intervals was taken $[(y_{max} - y_0)/N]$ where y_{max} = total fall distance, y_0 = initial position

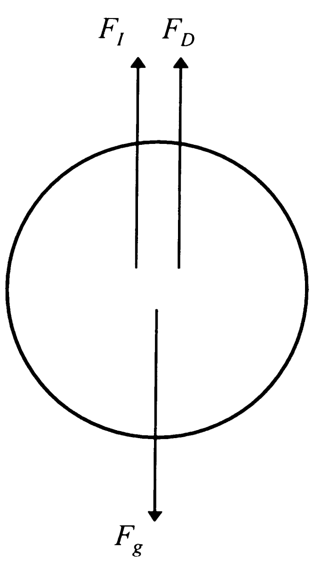

Figure 1–Forces acting on a falling sphere.

and N = number of iterations. The fall time for each individual interval was calculated using the inverse of equation 5. The total fall time t_{end} was obtained by summing over the intervals. In formula form:

$$t_{end} = \sum_{i=1}^{N} \left\{ C_{1_i} + \frac{1}{\sqrt{gK_i}} \operatorname{arccosh} \left[e^{K_i\left(\left(\frac{y_{max}-y_0}{N}\right) - C_{2_i}\right)} \right] \right\} \quad (8)$$

The C_D value for each interval was computed by using equations as assessed by von Zabeltitz (1967) for the transitional flow region. In this publication the regions overlap which leads to two C_D values for a single Reynolds number. This problem was dealt with by calculating the intersects of two overlapping curves and taking these intersects as the new interval markers being Re = 22.1, 2084, and 4000. The following formulas were used in these intervals:

$$Re < 22.1 \qquad C_D = \frac{21}{Re} + \frac{6}{\sqrt{Re}} + 0.28 \quad (9)$$

$$22.1 < Re < 2084 \qquad C_D = \frac{26.4}{Re} + \frac{4.87}{\sqrt{Re}} + 0.276 \quad (10)$$

$$2084 < Re < 4000 \qquad C_D = \frac{18.5}{Re} + \frac{6.23}{\sqrt{Re}} + 0.25 \quad (11)$$

$$4000 < Re \qquad C_D = 0.3531 \quad (12)$$

Equipment

The fall tests were carried out in sealed tube 15.83 m high, which is shown in figure 2. On top of the tube a distributor was mounted which releases individual particles on command. The fall time was measured between two sensor points. The sensor that started the timer, a photo voltaic transducer, was mounted 59 mm below the distributor. The sensor that stopped the timer consisted of a stainless steel fall plate on which the particles impacted. This plate had a piezo-electric transducer attached to it, that converted sound waves into electric charge. A charge amplifier was used to obtain a voltage signal.

Verification of Theory

The model was developed to explain the fall times of particles based on the assumption of perfectly spherical particle shapes. Therefore the validation of the model was carried out by using near perfectly spherical plastic particles. The plastics used were PTFE (Polytetrafluorethylene), POM (Polyacetal), and PP (Polypropylene), because of their density being in the range of commonly used fertilizer materials. Table 1 shows the comparison of plastics and fertilizer densities.

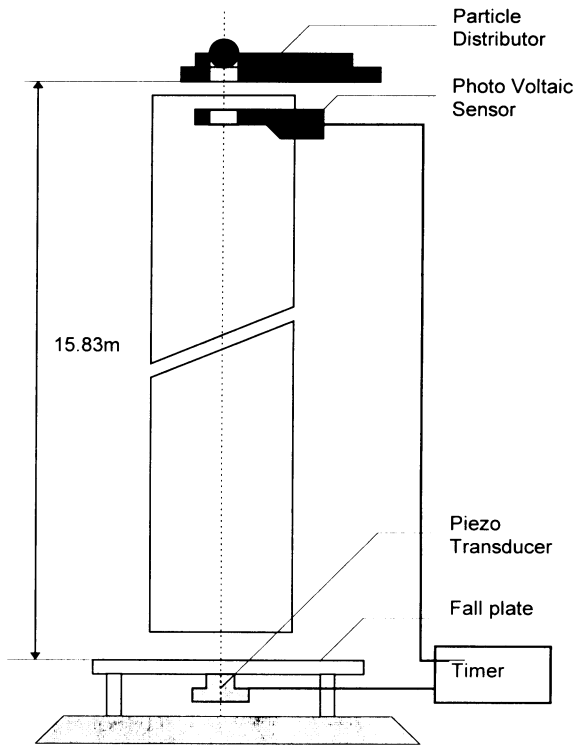

Figure 2–Fall test arrangement.

Table 1. Plastics used for validation

Plastic	Density (kgm^{-3})	Counterpart (kgm^{-3})	
PTFE	2150	NPK	2089
		Potassium	2033
POM	1410	Urea	1330
PP	903	Manure pellets	980

Table 2. Verification of theory

	Nominal Diameter (mm)	Measured Diameter (m)	Measured Weight (mg)	Fall Time Measured (s)	Fall Time Model (s)	Error (%)
PTFE						
	2	2.00	9.0	2.1738	2.1800	0.29
	3	2.98	29.6	1.9860	1.9872	0.06
	4	3.98	71.7	1.8978	1.8952	−0.14
	5	5.01	142.0	1.8472	1.8449	−0.13
	6	5.95	237.0	1.8211	1.8176	−0.19
POM						
	2	2.04	6.3	2.4241	2.4279	0.16
	3	3.00	20.0	2.1443	2.1447	0.02
	4	4.03	48.5	2.0053	2.0043	−0.05
	5	4.98	89.0	1.9410	1.9393	−0.09
	6	5.96	158.0	1.8869	1.8852	−0.09
PP						
	2	2.01	4.2	2.9024	2.7748	−4.40
	3	2.99	12.5	2.4578	2.4259	−1.30
	4	3.98	29.4	2.2126	2.2090	−0.16
	5	5.03	59.3	2.0849	2.0790	−0.28
	6	6.03	98.1	2.0201	2.0171	−0.15

The validation of the model was performed by dropping spheres of 2 to 6 mm nominal diameter and measuring their fall times. The diameter of the spheres was determined using a micrometer with a resolution of 0.01 mm. The masses were obtained, using an analytic balance with a resolution of 0.1 mg. The results of the validation fall test are shown in table 2. The fall times predicted by the model were quite close to the measured fall times. The maximum error of 4.4% was observed for a PP particle with a diameter of 2 mm. It must be realized however that this is the smallest particle of the lightest material, so relative errors in the measurement of diameter and mass become quite significant. For PTFE the errors were within 0.29%, and the errors for POM were less than 0.16%.

Table 3. True densities of fertilizer materials

Sieve Fraction (mm)	CAN 27 N True Density (kg/dm³)	NPK 12-10-18 True Density (kg/dm³)	Potassium 60 True Density (kg/dm³)
1-2	1.6737*	1.4911*	1.8522*
2-2.36	1.8673	2.0104	2.0436
2.36-2.8	1.8686	2.0928	2.0190
2.8-3.35	1.8507	2.0777	2.0251
3.35-4	1.8541	2.0982	2.0003
4-4.75	1.8600	2.0867	2.0215
> 4.75	1.1717*	1.9776	1.7525*
Average	1.8601	2.0889	2.0326

* Samples were too small to obtain an accurate estimate in Plethysmograph.

Materials and Methods

Three fertilizer materials were selected for their specific geometric properties: Calcium Ammonium Nitrate (CAN 27 N), Nitrate Phosphorus Potassium (NPK 12-10-18), and Potassium (Potassium 60). At first the bulk mass was sieved in fractions in screen sizes: 4.75, 4, 3.35, 2.8, 2.36, 2, and 1 mm. From each sieve fraction a sample was taken for weighing. Also from the same sample the volume was measured by means of plethysmography (Gundlach, 1980). The true density for each material was computed by taking the ratio of mass and volume, which is represented in table 3. The portion of particles smaller than 2 mm and larger than 4.75 mm was so small that their volume estimate could not be accurately determined with the plethysmograph. Therefore, their values were not used in the computation of the true density.

The size distribution of the fertilizers was obtained by measuring the mass of sieve fractions and is shown in figure 3. For the fall test 100 particles were selected randomly from each sieve fraction, leading to 700 particles per fertilizer material. Each particle was weighed individually using a "Sartorius 2001 MP2" electronic balance with a resolution of ±0.1 mg. Since the true density of each material was known the particle's 'corresponding' diameter (d_c) was computed by regarding the particle as a sphere with a known mass and density, using:

$$d_c = \sqrt[3]{\frac{6}{\pi}\left(\frac{m}{\rho_p}\right)} \quad (13)$$

where

d_c = 'corresponding' particle diameter (m)
m = mass (kg)
ρ_p = particle true density (kgm^{-3})

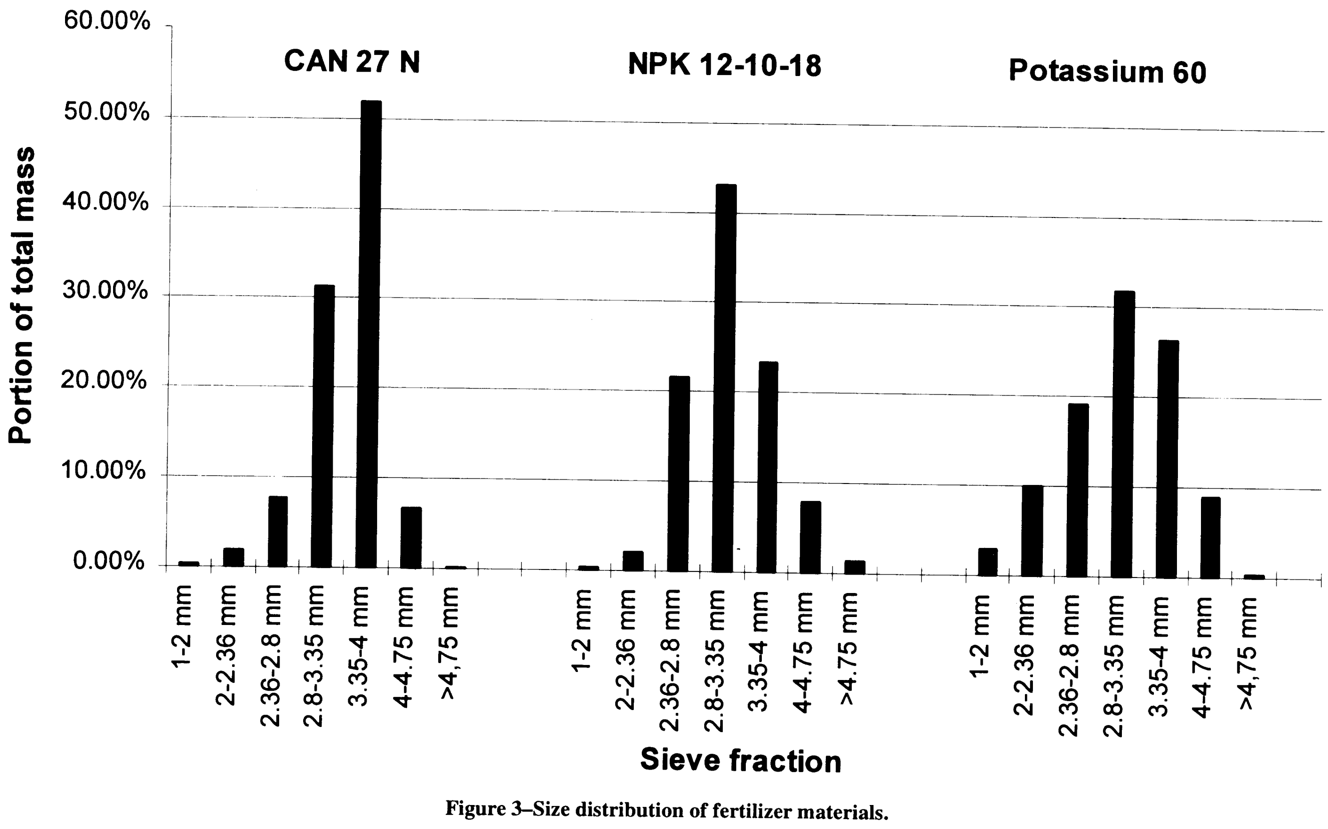

Figure 3–Size distribution of fertilizer materials.

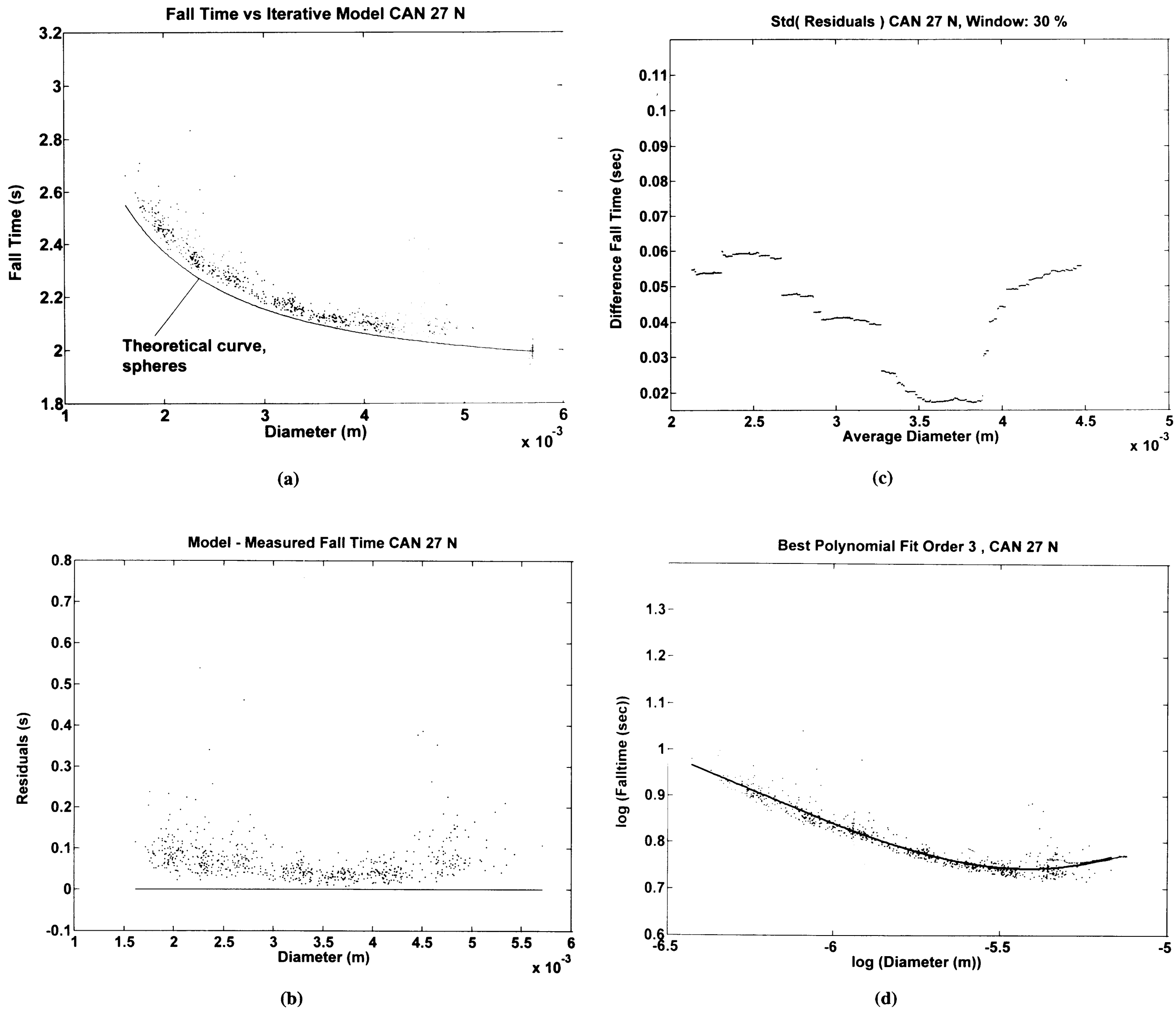

Figure 4–Fall test data and acquisition results CAN 27 N.

The fall times of the particles were measured with an accuracy of 0.1 μs. Each particle was dropped three times unless it broke during the first or second time. When particles contacted the wall of the fall tube the data were also neglected. The recorded fall times were stored in files under control of an ASYST program. The data were analysed by a custom MatLab (1995) program that compared the measured fall times with the theoretical according to the iterative model.

RESULTS

Results of the fall tests are shown in figures 4, 5, and 6 for CAN 27 N, NPK 12-10-18, and Potassium 60, respectively. The three figures each are composed of **raw data, residuals, variability,** and **descriptive model**. In the raw data plot (upper left) the measured and modeled fall times are plotted versus the 'corresponding' diameter. In the residuals plot (upper right plot) the differences between measured and modeled fall times are given versus the 'corresponding' diameter. In the variability plot (lower left) the standard deviation was computed for the residual population in a 'window' while sliding it along the diameter axis. Thus an estimate for the variability in a moving average sense was obtained. In the descriptive model plot (lower right), a third-order polynomial function was fitted on the log-log representation of the raw data. The fall times of fertilizer particles will differ from those of spheres with a 'corresponding' diameter because their aerodynamic drag coefficient is influenced by their non sphericity, surface texture and possible spinning effects. Hofstee (1993) observed that the fall time of an irregular fertilizer particle was always longer than the fall time of a theoretical spherical particle with a 'corresponding' diameter. This means that a deviation from the perfect spherical shape will always result in an increase in aerodynamic friction. This increased friction plays a role during the whole fall process and therefore the difference between measured and theoretical fall times will be

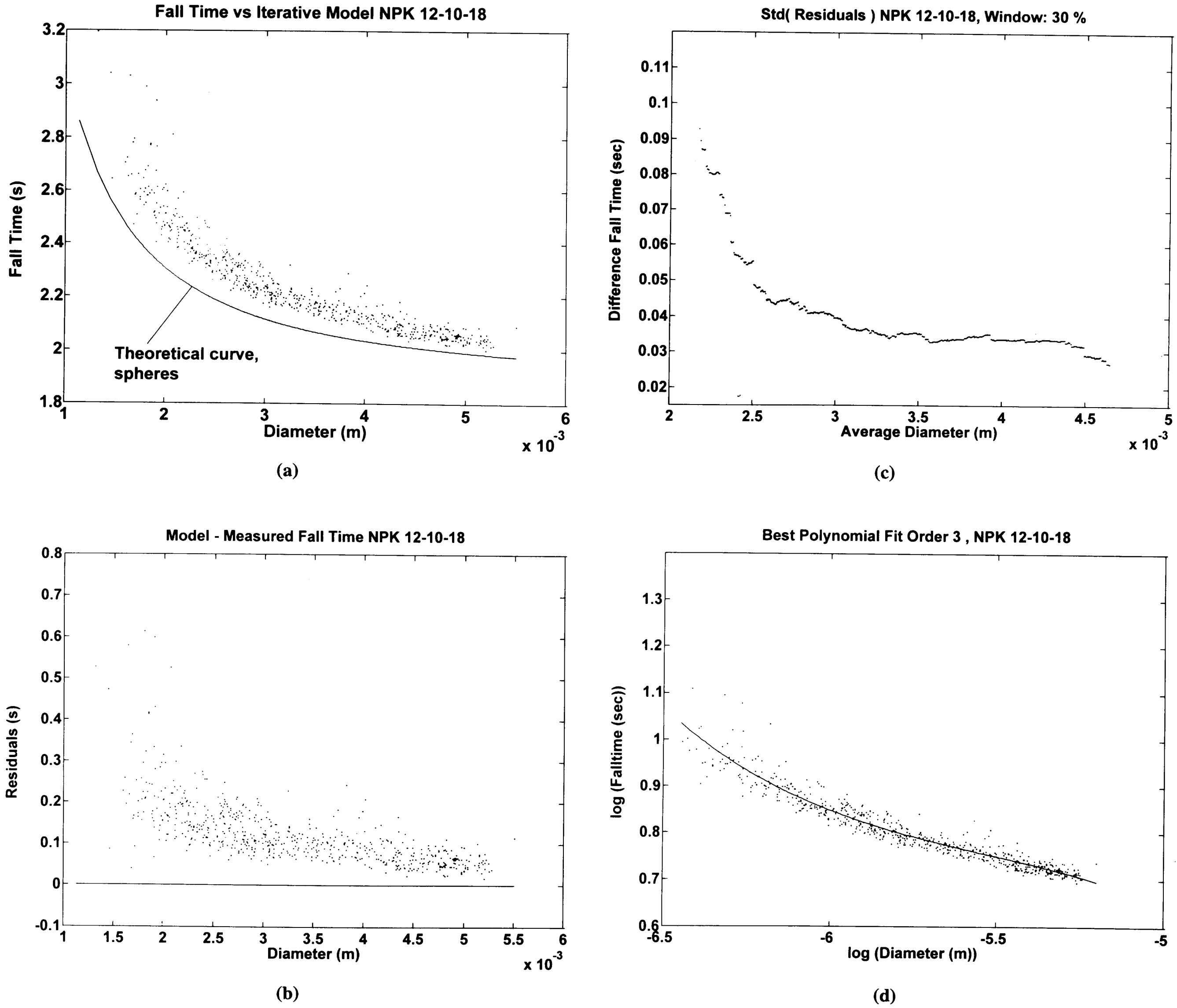

Figure 5–Fall test data and acquisition results NPK 12-10-18.

proportional to the fall time itself. In the test results this should be expressed in a band of fall times which range broadens for the smaller particles. This would show up as a monotonously decaying line in the variability plot.

In figure 4 the fall test results of CAN 27 N are presented. The raw data show a band of particles that have slightly longer fall times than predicted by the theoretical line. In the diameter region 3 to 4 mm the band is of constant width as can be seen in the residual plot. In the region of smaller diameters, the band broadens as expected. In the region of larger diameters (>4.5 mm), however, all particles have a distinctly longer fall time than predicted. The variability plot shows a sudden increase for these particles. This phenomenon might be caused by the fact that during the production process large particles are broken and fed back in the process which gives very irregular particles. The behavior described here has led to the decision to select CAN as a representative material to study non-sphericity effects which will be published in Walker et al. (1996). The descriptive model plot shows a line that is linear and curves up at higher diameter values caused by the broken particle phenomenon.

The data for NPK 12-10-18 is presented in figure 5. Again, the deviations band broadens for smaller diameters. From the standard deviations it can be seen that this effect seems to increase for particles smaller than 2.5 mm. In the descriptive model plot, the linear part of the exponential fit curve covers a major portion of the data. Figure 6 shows the data for Potassium. The initial expectations as described earlier were best met by Potassium. The residuals show a cloud whose range is decreasing with the diameter. The variability plot shows an almost linear decay. Furthermore the data can be approximated by an exponential model quite well as can be seen from the virtually straight line in the descriptive model plot.

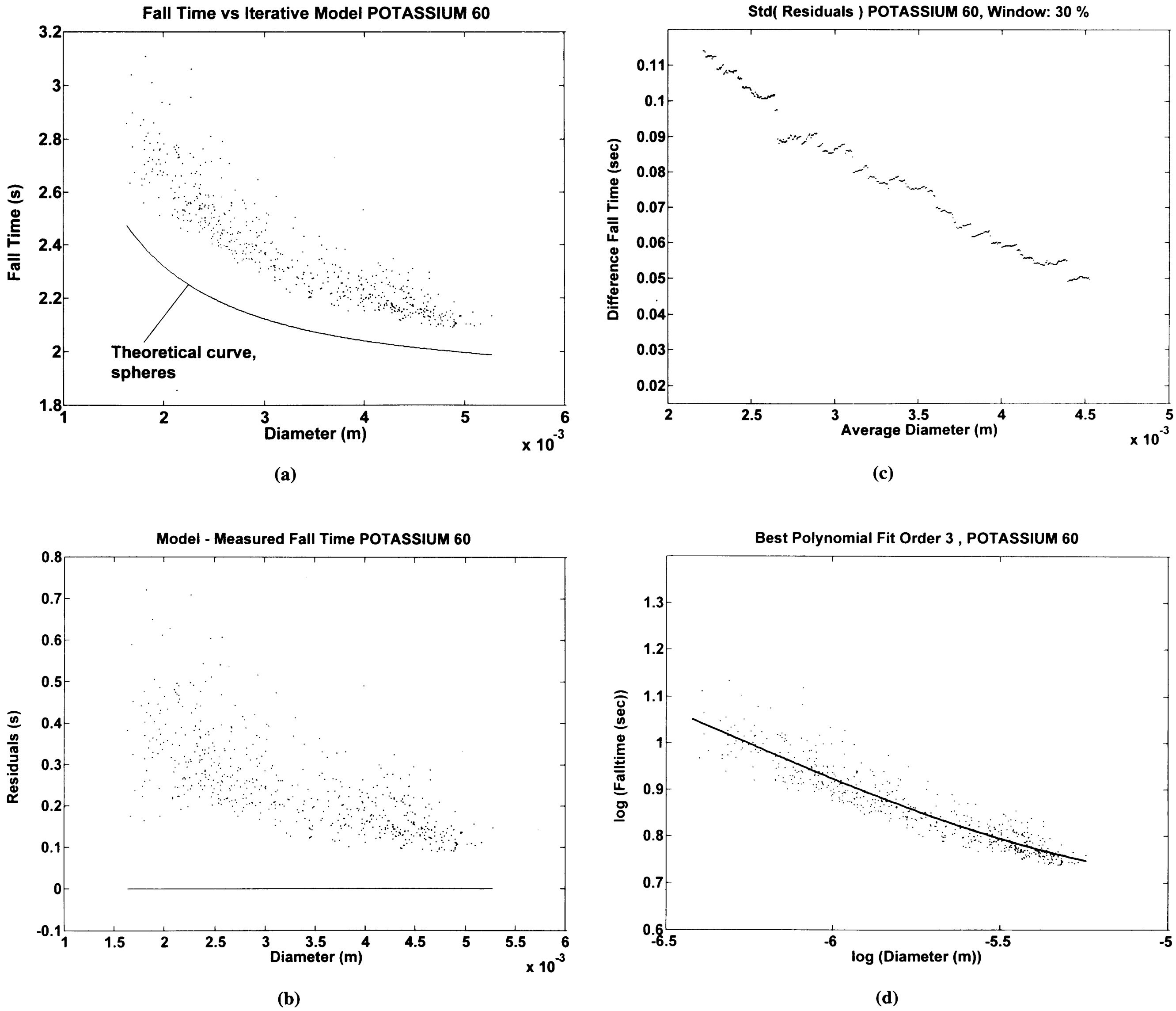

Figure 6–Fall test data and acquisition results Potassium 60.

Q-Factors

Hofstee (1993) suggested q-factors as a method to compensate for the effects in aerodynamic behavior of fertilizer particles due to their non-spherical nature. The procedure is as follows: First the 'corresponding' diameter (d_c) of a fertilizer particle is computed by regarding it as a sphere with a known mass and density. This particle is dropped and it's fall time recorded. This fall time corresponds to a unique perfectly spherical particle, according to the theory described earlier. The diameter of this perfect sphere is called the 'equivalent' diameter (d_e). An example for an arbitrary CAN particle is given in figure 7. The datapoint is emphasized by the circle around it.

The q-factor is now defined as $q = d_e/d_c$. In other words, the q-factor is a dimensionless number that relates an irregular fertilizer particle to a perfectly spherical particle, based on equalized fall times. The q-factors for each material per sieve fraction are shown in table 4. The standard deviations of the q-factors are listed under 'σ'.

Conclusions and Further Research

The data from the fall tests show that the aerodynamic behavior of fertilizer particles is significantly influenced by their non-spherical nature. As can be expected intuitively, the effects were found to be stronger for rougher particles such as Potassium 60.

A theoretical model for the aerodynamic behavior of perfectly spherical particles was developed. This model was validated by dropping plastic spherical particles made of PTFE (Polytetrafluorethylene), POM (Polyacetal), and PP (Polypropylene) of diameter 2, 3, 4, 5, and 6 mm. The validation showed that 13 out of 15 observations were accurate within 0.29%. The remaining two predicted fall times were within a 5% of the measured fall times which was probably due to errors in measurement of diameter and/or mass since they were the smallest particles of the least dense material.

The q-factors have shown to be practical way to express the non-sphericity of fertilizer particles. The obtained values discriminate the three materials quite well. For the

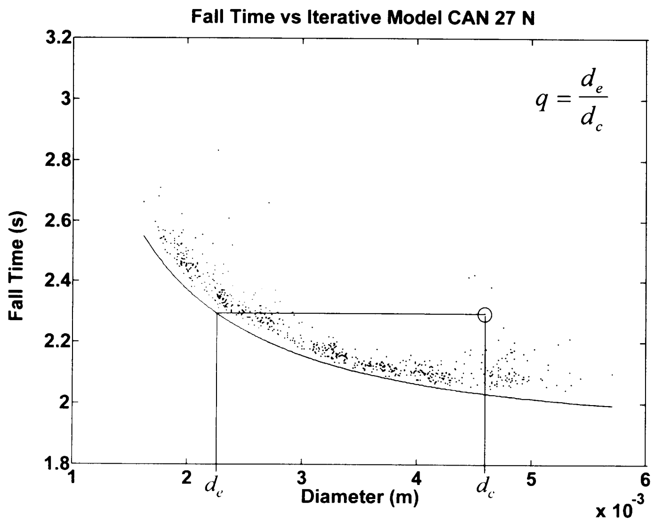

Figure 7–Procedure for q-factor computation.

Table 4. q-factors of fertilizer materials

Sieve Fraction (mm)	CAN 27 N		NPK 12-10-18		Potassium 60	
	q (1)	σ (1)	q (1)	σ (1)	q (1)	σ (1)
1-2	0.8922	0.0372	0.7696	0.0882	0.6829	0.0691
2-2.36	0.8865	0.0591	0.7773	0.0506	0.6474	0.0650
2.36-2.8	0.8660	0.0580	0.7735	0.0506	0.6361	0.0600
2.8-3.35	0.8740	0.0427	0.7690	0.0517	0.6221	0.0579
3.35-4	0.8686	0.0400	0.7364	0.0559	0.6149	0.0644
4-4.75	0.7853	0.0941	0.7392	0.0595	0.5909	0.0565
> 4.75	0.6858	0.0836	0.7336	0.0541	0.6186	0.0372

sieve fractions that represent the bulk of the materials the q-factor was found to be approximately 0.87 ($\sigma = 0.04$) for CAN, 0.75 ($\sigma = 0.05$) for NPK, and 0.62 ($\sigma = 0.06$) for Potassium.

The fall test has shown to be a valuable instrument for studying aerodynamic properties of fertilizer particles. The system is simple, reliable and easy to maintain. A large scale use of a fall test is limited however by the availability of an environmentally controlled fall tower of several meters height.

In previous work other factors were found to influence the aerodynamic behavior of fertilizer particles such as the spinning of particles in an airflow which increases the drag force. In order to isolate the non-sphericity of fertilizer particles among other variables that might influence the aerodynamic behavior, further study is necessary in which a measure for non-sphericity (shape factor) is correlated with fall times.

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