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Estimation of the flow rate of free falling granular particles using a Poisson model in time

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A generic method to estimate the flow rate (number of particles passing per unit of time) of free falling granular particles in a tube is presented. This principle could be applied in pneumatic planters, fertiliser distribution, yield monitoring of grains and fruits and various industrial applications. The flow of particles in a tube was observed as an intermittent succession of clumps of particles separated by spacings among them. Since the clumping effect prohibits counting individual particles, the hypothesis was that the flow constitutes a random arrival process in time. In other words, the process of particle arrivals at the sensor was assumed equivalent to classical Poisson driven arrival processes from queueing theory, such as telephone calls arriving independently at a helpdesk. The assumption of a Poisson process allows for simple flow rate estimation, since according to theory, the flow density of such a process is equal to the reciprocal value of the mean of the spacing time intervals, which can be measured.

An optical single interruption plane sensor was used to measure the time intervals during which clumps and spacings pass. This sensor suffers from inherent errors such as defocus and uncertain optical switching behaviour. Therefore, the sensor was characterised by equalising measured quantities with their theoretical equivalents. This implies that the estimated mean flow rate must be equal to the theoretical mean flow rate, however, the variability and extreme values among experiments indicate the usefulness and appropriateness of the method. To test the validity of the Poisson model assumption, 30 experiments were conducted in which 4000, 4.5 mm identical spherical particles were dropped from a funnel into a fall tube.

To assess the performance of the method, it is not possible to compare measured flow rates with reference counterparts, since there are none. However, the initial number of particles per experiment is known and, therefore, this number was estimated using measurements. After characterisation, the original number of 4000 particles per experiment was estimated at 4000 with a standard deviation of 44 (1.1% coefficient of variation) among 30 datasets. The extreme values of the estimations were 4092 (+2.3% error) and 3930 (−1.8% error), respectively.

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Nomenclature

N	original number of particles in experiment
N_T	total number of clump/spacing pairs counted during experiment
$TC_{i,Theor}$	theoretical interruption interval of ith clump, s
$TS_{i,Theor}$	theoretical interruption interval of ith spacing, s
$\overline{TC}_{Theor}$	theoretical mean of clump interruption intervals over N_T measurements, s
$\overline{TS}_{Theor}$	theoretical mean of spacing interruption intervals over N_T measurements, s
λ_{Theor}	theoretical flow density, s^{-1}
$\overline{TC}$	measured mean of clump intervals over N_T measurements, s
$\overline{TS}$	measured mean of spacing intervals over N_T measurements, s
$\hat{\lambda}$	estimator for flow density, s^{-1}
TT	total duration of experiment, s
$\hat{N}$	estimator for number of particles in experiment
b	correction factor representing errors in the sensor

1. Introduction

Many materials in agriculture are transported and applied in granular form and their instantaneous flow rate is an important measure. For instance, during harvesting the flow of grains is measured using yield monitors, one of the enabling technologies of Precision Agriculture. In ground-based and aerial fertiliser application, there is a need for an accurate flow rate sensor for use as a feedback indicator for either a human operator or a closed loop automatic flow control system allowing variable rate application. The same problem arises in pneumatic planters, where seeds and fertiliser granules are transported in tubes from a central hopper to soil engaging tools. These systems currently operate without feedback control of flow rate, which limits their functionality in variable rate scenarios. The system described here is a strong candidate for application in these areas since it gives a real time estimate of the instantaneous flow rate in particles per time unit. This measurement can be translated into a mass flow rate by multiplying the flow rate by the average particle mass.

Flow rate measurement of granular particles has traditionally been based on indirect methods such as energy absorption, capacitance, magnetism, optics and acoustics (Yong, 1996). In contrast, here it was assumed that the particles falling from a funnel into a tube pass a fixed location in the tube (that is, a location where a sensor is present) according to the principles of a Poisson process. The Poisson process is widely used to describe events that happen discretely and randomly in time (Taylor and Karlin, 1998). Typical examples of discrete random variables described by a Poisson process are the number of telephone calls received by a helpdesk in a given period of time or the number of stars above a certain level of brightness in a particular patch of sky. Characteristics of a homogeneous Poisson process are (1) that the time intervals among the random events (such as telephone calls) are exponentially distributed, and (2) that the mean number of events per time unit (termed density) is equal to the reciprocal value of the mean of these inter-event intervals where an event corresponds to a particle passage. In this research it was not possible to measure inter-particle intervals, but merely inter-clump intervals. However, the mean of the inter-particle intervals is assumed equal to the mean of the inter-clump intervals and, therefore, it suffices to obtain the flow rate in particles per time unit.

The fact that the flow density theoretically can be obtained solely from the mean inter-clump (spacing) intervals indicates that the method is independent of the particle diameter distribution and theoretically works for any material, contingent upon a Poisson arrival process being present. This has not been proven in practice, since in this research only identical spherical particles were used.

Fig. 1 shows an image of 4.5 mm identical particles free falling from a funnel through a tube (from left to right). When regarded laterally, the flow indeed resembles an intermittent succession of overlapping particles, termed clumps with spacings among them. To estimate the flow rate, an accurate count of the number of particles passing an arbitrary location along the tube is needed, but as the image shows it is impossible to count individual particles owing to the clumping effect. Unless the flow is made extremely sparse, any radiation interruption sensor with a sensing plane perpendicular to the tube

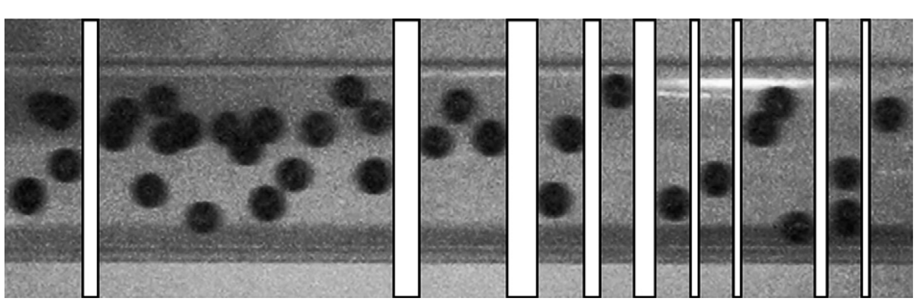

Fig. 1 – Spherical particles (4.5 mm) accelerating in a tube from left to right. The bars indicate spacings among the clumps.

axis will inevitably count the number of clumps (and an equal number of spacings among them) rather than individual particles. The challenge is now to estimate the number of particles contained in the clumps as a measure of flow rate based on the measured clump and spacing interval times. For this purpose, a sensor is needed that measures the times in which the sensor is vacant during passage of a spacing (indicated by the vertical bars in Fig. 1) and times that the sensor is occupied during passage of a clump. The sensor used was developed in earlier research (Grift and Hofstee, 1997).

The fact that the number of particles passing per time unit (flow rate) can be estimated using solely the spacings among them is counter-intuitive. A typical engineering mindset is that the highest accuracy of measurement is obtained when the process is perfectly controlled, for example by counting singulated particles. The method presented here implies the opposite: the most accurate flow rate measurement can be obtained when the flow is completely random. In fact, it is beneficial to use randomisers (such as a funnel) to stimulate the random flow formation process.

The objective of this research was to estimate the flow rate of free falling granular particles, and *en passé* validate the assumed Poisson driven arrival model.

2. Materials and methods

To illustrate the clump formation, Fig. 2 shows a hypothetical experiment with 20 identical particles that form clumps and spacings among them. Here, owing to the clumping process, 8 clump/spacing pairs (N_T) are created.

After the experiment the number of clump/spacing pairs N_T is known without error. In addition, the sum of the clump intervals in s and spacing intervals in s is known accurately. The theoretical mean of the clump intervals $\overline{TC}_{Theor}$ in s can be computed as follows:

$$\overline{TC}_{Theor} = \frac{1}{N_T}\sum_{i=1}^{N_T} TC_{i,Theor} \tag{1}$$

here, $TC_{i,Theor}$ is the theoretical *i*th clump interval in s. Analogously, the theoretical mean of the spacing intervals $\overline{TS}_{Theor}$in s can be obtained:

$$\overline{TS}_{Theor} = \frac{1}{N_T}\sum_{i=1}^{N_T} TS_{i,Theor} \tag{2}$$

here, $TS_{i,Theor}$ is the theoretical *i*th spacing interval in s. The total duration of the experiment TT in s is equal to the sum of the clump and spacing intervals, which can be written as follows:

$$TT = \sum_{i=1}^{N_T}(TC_{i,Theor} + TS_{i,Theor}) = N_T(\overline{TC}_{Theor} + \overline{TS}_{Theor}). \tag{3}$$

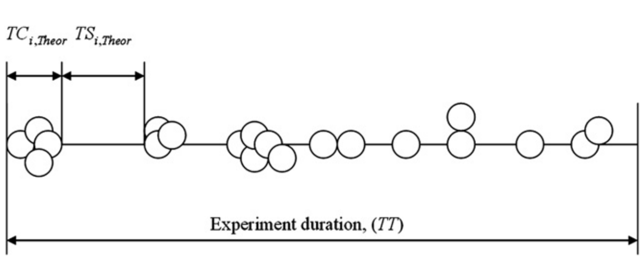

Fig. 2 – Example experiment with N = 20 initial number of particles. The number of clump/spacing pairs $N_T = 8$.

Hall (1988) showed that whilst assuming a stationary Poisson flow model in time, the theoretical flow density λ_{Theor} in particles per s, is equal to the reciprocal value of the mean spacing interval times $\overline{TS}_{Theor}$in s or:

$$\lambda_{Theor} = \frac{1}{\overline{TS}_{Theor}}. \tag{4}$$

The flow rate in experiments is not known *a priori* and, therefore, to assess the performance of the flow rate measurement method, it is simpler to compare the estimated number of original particles with the true number (4000). An estimate of the original number of particles in the experiment $\hat{N}$ can be obtained by multiplying the flow density λ_{Theor} in s^{-1}, by the total experiment duration TT in s. Combination of Eqs. (3) and (4) yields:

$$\hat{N} = \lambda_{Theor}TT = \frac{1}{\overline{TS}_{Theor}}N_T(\overline{TC}_{Theor} + \overline{TS}_{Theor}) = N_T\left(\frac{\overline{TC}_{Theor}}{\overline{TS}_{Theor}} + 1\right). \tag{5}$$

The flow density can subsequently be computed by dividing the estimated number of particles by the accurately measured experiment duration TT in s or:

$$\hat{\lambda} = \frac{\hat{N}}{TT}. \tag{6}$$

As is clear from Eq. (5), the estimate of the original number of particles in experiments depends on the number of clump/spacing pairs N_T, which is counted without error, as well as the ratio of the mean clump and spacing interval times in s. The latter are measured by the sensor, and contain an error. To correct for this error, sensor characterisation was carried out as described in Section 2.2.

2.1. *Simulation example of a Poisson driven flow experiment*

To illustrate the formation of clumps and spacings in a Poisson flow, Fig. 3 shows the sorted clump and spacing interval times of a Poisson driven flow experiment where 4000 identical 4.5 mm diameter particles pass a sensor that measures the clump and spacing interruption time intervals. The flow rate was set to 100 particles per s and a speed of 1 m s^{-1} was assumed. At this flow rate, among the 4000 initial particles, 2552 clump/spacing pairs were counted. Among the 2552 clumps there are 1627 with a time interval of 4.5 ms (bottom graph). These represent single particles with a diameter of 4.5 mm. The remaining 925 clumps contain more than one particle. Note also that the sorted spacing times (top graph) resemble an exponential distribution from which the flow rate can be directly computed using Eq. (4). As the mean of the spacing times was 9.9566 ms, the flow rate was computed as $\lambda_{Theor} = 1/\overline{TS}_{Theor} = 1/9.9566\ \text{ms} = 100.0435$ particles per s. To estimate the initial number of particles in the simulated experiment (4000) Eq. (5) was used. The mean of the clump times was 5.654 ms, which gives an estimate of the initial number of particles being

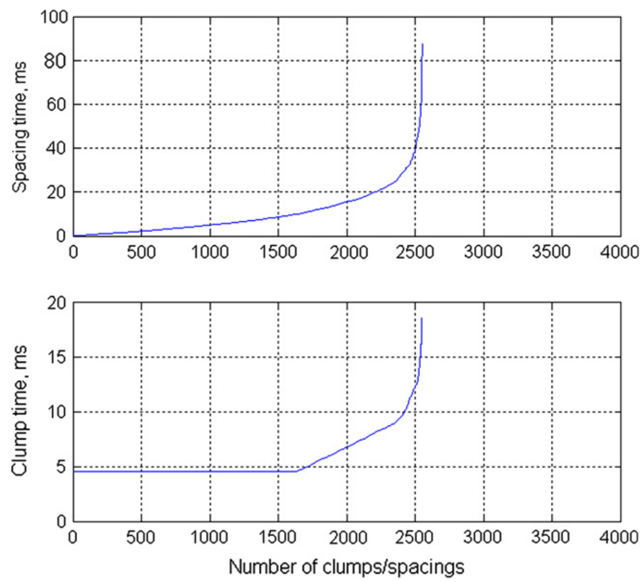

Fig. 3 – Simulated sorted clump and spacing times of a Poisson driven flow experiment using 4000 identical particles with a diameter of 4.5 mm, at a density of 100 particles per s, resulting in 2552 clump/spacing pairs. Note that the clump times (bottom graph) show 1627 single particles; the remaining 925 clumps contain more than one particle. Note also that the spacings times (top graph) have an exponential distribution.

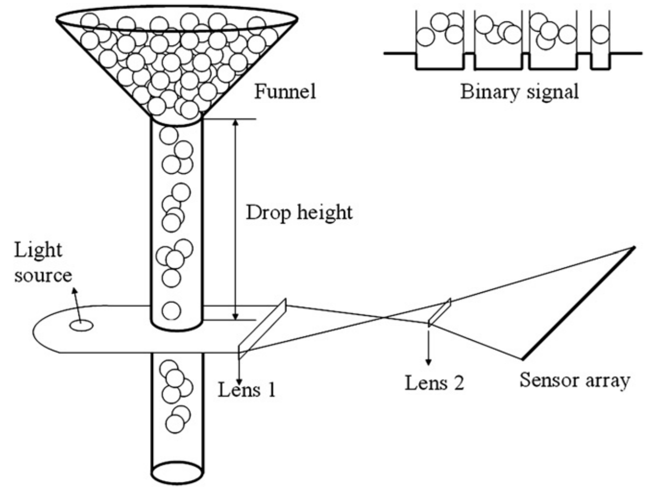

Fig. 4 – Single-layer optical photo interruption mechanism. The image of clumps is projected onto the sensor array, which consists of 30 digital optical switches (OptoSchmitt, SDP8601, Honeywell, Scotland, UK). All 30 OptoSchmitts are connected in a logical AND function, essentially forming an optical plane consisting of 30 light sensitive lines placed side by side at a distance of approximately 0.63 mm. The lenses magnify the image of the particles by a factor of 8, which allows the detection of small particles (up to 1 mm) using relatively large sensors (5 mm width). The output of the sensor array is shown at the top right.

$\widehat{N} = N_T(\overline{TC}_{Theor}/\overline{TS}_{Theor} + 1) = 2552(5.654\ \mathrm{ms}/9.9566\ \mathrm{ms} + 1) =$ 4001 particles.

2.2. *Sensor characterisation*

Fig. 4 shows the principle of a single-layer photo interruption device. The sensor array consists of a series of 30 digital optical sensors termed 'OptoSchmitts' (SDP8601, Honeywell, Scotland, UK). All OptoSchmitts are placed in a logical AND function, which means that the array output will respond to being blocked by a clump by becoming low when one or more of the 30 OptoSchmitts are blocked by the particles' shadows. This method effectively creates a photo-sensitive optical plane which translates the three dimensional clumps into a one dimensional clump length. The sensor contains two lenses, which magnify the image of the clumps onto the sensor array. The magnification allows small particles (up to 1 mm diameter) to be detected using the OptoSchmitts, which are spaced 5 mm apart. The clump and spacing time intervals were measured using a counter/timer board with a clock rate of 20 MHz (PCI-6601, National Instruments, TX, USA).

Three sources of error are present in the sensor being (1) defocus, since only particles that fall in the centre of the funnel are projected in focus, (2) error owing to the unknown exact switching point of the sensor array, which depends on the light intensity and (3) error owing to off-centre particle passage caused by the sensor array consisting of 30 discrete OptoSchmitts rather than a continuous array. The net effect of these errors is that although the sum of the clump and spacing intervals during the experiment is measured accurately, the clumps may be measured excessively long and the spacings excessively short or *vice versa*. To accommodate for this, the model proposed for the measurement is a variation on Eq. (5) where a parameter is inserted before the ratio of the measured mean clump intervals $\overline{TC}$ and spacing intervals $\overline{TS}$. Note that the measured mean clump and spacing time interval symbols have no subscript

$$N = N_T\left(b\,\frac{\overline{TC}}{\overline{TS}} + 1\right). \tag{7}$$

The value of b was now estimated by equalising the theoretical Eq. (5) and measurement Eq. (7) and solving for the parameter b as follows:

$$b = \frac{\overline{TC}_{Theor}}{\overline{TS}_{Theor}}\,\frac{\overline{TS}}{\overline{TC}}. \tag{8}$$

The value of b was determined as 0.67 for all 30 datasets using Eq. (8), and its mean value among the datasets was adopted as a sensor characteristic constant. This value would ideally be unity, but it is not owing to the sensor suffering from errors: Even if a better sensor with a value of b closer to unity would be developed, the need for sensor characterisation would remain. The value of 0.67 should not be misconstrued as indicating a non-Poisson driven flow. Evidence that the flow indeed constitutes a Poisson driven arrival process can be found in the companion paper (Grift and Crespi, 2008).

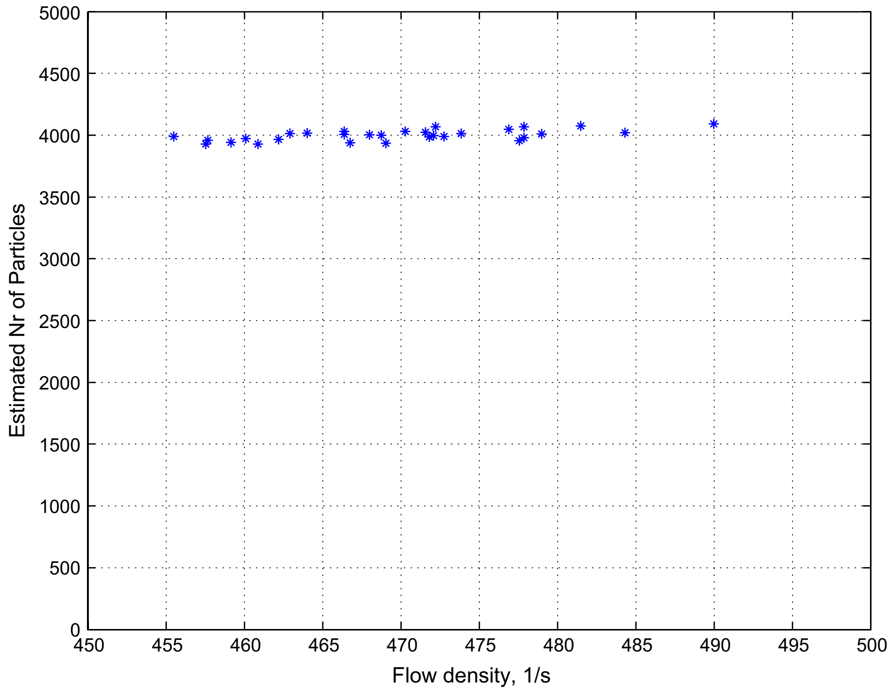

Fig. 5 – Estimated number of particles from experiments with 4000 identical 4.5 mm particles at densities ranging from 455 to 490 particles per s. The mean of the estimates was 4000 with a standard deviation of 44, and a coefficient of variation of 1.1%. The maximum and minimum values of the estimations were 4092 (+2.3% error) and 3930 (−1.8% error), respectively.

3. Results

Experiments were carried out using a funnel and fall tube arrangement as shown in Fig. 4. For each experiment, 4000 particles were dropped into the funnel in a swift motion after which the funnel emptied owing to gravity. The mean experiment duration among 30 datasets was 8.51 s, with a standard deviation of 0.12 s. The experiments were carried out in groups of 10, at three different drop heights.

Fig. 5 shows the estimated number of particles from the 30 experiments with 4000 particles using Eq. (7). The mean value among the experiments was 4000 (after characterisation), with a standard deviation of 44 (1.1% coefficient of variation). The maximum and minimum values of the estimations were 4092 (+2.3% error) and 3930 (−1.8% error), respectively. The values of the estimated flow densities ranged from 455 to 490 particles per s.

4. Conclusions

A method was devised to estimate the flow rate of particles free falling in a tube. The flow of particles was assumed a Poisson driven arrival regime, where particles arrive independently at a sensor. If the flow is indeed Poisson driven, according to theory, the desired flow rate is equal to the reciprocal value of the mean spacing time intervals among the clumps of particles.

The sensor contains errors and, therefore, it was characterised using the data. In this process, the measured flow rate was equalised with the theoretical counterpart. A sensor characteristic parameter which represents errors in the sensor was found to be 0.67, whereas the ideal value would be unity. The value of 0.67 indicates that the sensor measures clumps excessively long and spacings excessively short: it does not imply that the flow is not Poisson driven.

Instead of estimating the flow rate, which is neither known *a priori* nor measured directly, the original number of particles per experiment (4000) was estimated. Among 30 experiments where particles fell from three different fall heights, the initial number of particles was estimated at 4000 (owing to characterisation) with a standard deviation of 44 (1.1% coefficient of variation). The maximum and minimum values of the estimations were 4092 (+2.3% error) and 3930 (−1.8% error), respectively. The estimated flow densities ranged from 455 to 490 particles per s.

This research shows that the method presented holds promise for a generic flow rate measurement of any granular particle flow that can be regarded as a Poisson arrival process. However, the fact that the method works for any material has not been proven in this research since only identical particles were used. In addition, the sensor characteristic value of 0.67 is theoretically valid for any material if the flow indeed constituted a Poisson arrival process, however, this research has not confirmed this. Further experimentation with non-identical particles will be needed to investigate this.

REFERENCES

Grift T E; Hofstee J W (1997). Measurement of velocity and diameter of individual fertiliser particles by an optical method. Journal of Agricultural Engineering Research, **66**(3), 235–238. doi:10.1006/jaer.1996.0128.

Grift T E; Crespi C M (2008). Estimation of mean particle diameter in free fall granular particle flow using a Poisson model in space. Biosystems Engineering, **101**(1), 28–35.

Hall P (1988). Introduction to the Theory of Coverage Processes. Wiley, New York.

Taylor H M; Karlin S (1998). An Introduction to Stochastic Modeling (Third ed.). Academic Press, San Diego.

Yong Y (1996). Mass flow measurement of bulk solids in pneumatic pipelines. Measurement Science and Technology, **7**, 1687–1706.