Mass Flow Measurement of Granular Materials in Aerial Application — Part 1: Simulation and Modeling

T. E. Grift

Abstract. *The mass flow of granular particles in an aerial spreader duct was regarded as a sequence of cluster passage events. At low flow densities, the total mass per time unit could be estimated by measuring the diameter (length) of each individual particle passing a sensor and accumulating the associated masses. At higher flow densities, the lengths of clusters would be measured rather than the lengths of particles. However, because of overlapping, the cluster lengths cannot simply be accumulated. The total length of a cluster is always smaller than the lengths of the individual particles within it. Therefore, a reconstruction method is necessary to estimate the total length of the particles within a cluster from the measured cluster length. This reconstruction algorithm was developed using MatLab™ as a simulation tool and was called the "Exponential Estimator." Simulations were conducted for particles with 1) Identical diameters, 2) Uniformly distributed diameters, 3) Gaussian distributed diameters, and 4) Urea–distributed diameters. A simple universal relationship was discovered between the event ratio (the ratio between the original number of particles in an experiment and the number of measured clusters) and the flow density. This relationship was found to be independent of both the mean diameter of particles and the diameter distribution, which is of great importance when mass flows of fertilizer are involved. The flow density cannot be measured directly. However, another simple relationship was found between the flow density and the number of clusters in certain length categories, which can be measured on the fly. This relationship was found to be independent of the mean diameter of particles, but dependent on the diameter distribution. Combination of these two relationships led to the Exponential Estimator. It contains only a single material–specific constant for distributed–diameter particles. The simplicity and compactness of the discovered relationships indicate the possibility to derive the Exponential Estimator from theoretical principles. The simulation tool as developed here could be a valuable instrument for this purpose.*

Keywords. *Mass flow measurement, Clustering process, Aerial application, Fertilizer, MatLab™.*

Aerial application has been recognized for decades as an efficient tool to disperse fertilizer materials over large areas. Although modern airplanes make parallel swaths guided by sophisticated GPS equipment, sometimes "streaks" occur in the field. These areas of over–application or under–application have a variety of causes and potential effects on crop, soil, and the environment. Factors causing non–target application are poorly adjusted equipment, varying material properties (such as density and aerodynamic drag), aerodynamic "cleanliness" of the airframe, and varying ambient air conditions during spreading. Another important factor is that the spread characteristics of a spreader are highly sensitive to load changes. Aerial applicators usually calibrate their equipment once a year at calibration workshops (Gardisser and Walker, 1990). The spreading performance is evaluated at an average target rate. In their daily practice, however, pilots often vary the rate, for instance to compensate for head or tail wind. The rate is controlled by a simple lever–operated gate, but without a feedback mechanism, pilots must adjust the gate by intuition and experience, which can lead to spread patterns of unacceptable quality (Grift et al., 2000). A mass flow sensor, delivering a direct indication of the material output of the spreader, would give aerial applicators much more control over their work. In spraying applications, this kind of feedback is available in the form of a simple fluid flow meter, but for granular materials it does not exist.

If a mass flow sensor could be placed in each duct, then a predicted spread pattern on the ground could be computed based on a ballistic model and material–specific aerodynamic properties. This spread pattern prediction concept was first defined by Hofstee et al. (1994). The predicted pattern could be used to simulate consecutive overlaps and compute an optimal swath width, yielding the desired application rate at an acceptable uniformity. This "smart spreader" approach would enable the development of variable–rate equipment for aerial application of granular materials.

A digital opto–electronic device that measures the lengths of clusters of particles was developed by Grift and Hofstee (1997). Although the sensor measures the cluster lengths accurately, the measurement of mass flow requires a reconstruction algorithm that estimates the true mass flow from the measured cluster lengths. In this research, the clustering process itself was studied using simulation, in order to relate the sensor information to reliable estimates of the mass flow. Two conditions under which mass flows can be measured were defined:

Single–Particle Approach (SPA) — This approach is based on measuring the diameter of each individual particle in a mass flow regime. Each diameter is translated into a volume and multiplied by the true material density to yield a mass flow. Obviously, this method works only if no clustering occurs, such as under very low flow density conditions.

Mass Flow Approach (MFA) — This approach is a variation of SPA in which the lengths of clusters, instead of

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The author is **Tony E. Grift, *ASAE Member Engineer,*** Assistant Professor, Department of Biosystems Engineering, Auburn University, Auburn, Alabama. **Corresponding author:** Tony E. Grift, 213 Tom E. Corley Bldg, Auburn University, Auburn, AL 36849; phone: 334–844–3545; fax: 334–844–3530; e–mail: tegrift@eng.auburn.edu.

single particles, are measured and accumulated. Because of the clustering effect, the total length of the particles in a cluster needs to be reconstructed from the total length of clusters. The algorithm developed for this purpose is called the “Exponential Estimator.” It is based on the characteristic increase in cluster lengths at higher flow densities.

The objective of this study was to identify relationships that could be used to perform mass flow measurements of granular materials in aerial spreader ducts, based on measured cluster lengths. The experimental validation of the relationships is reported in Grift et al.(2001).

Problem Statement

If an experiment could be conducted in which an ideal sensor was used to measure the diameter of N identical particles individually, then the results could be represented as the top bar graph in figure 1. This regime can only exist under extremely low flow densities, when no overlapping of particles occurs. When the density is increased, particle overlap occurs. A significant number of particles would be measured with diameter D, but cluster lengths of two or more overlapping particles would also emerge. After sorting of the cluster lengths, the results would resemble the bottom bar graph of figure 1. The task of the reconstruction algorithm is now to relate the total number of measured clusters (E) to the original number of particles in the experiment (N) for any given flow density.

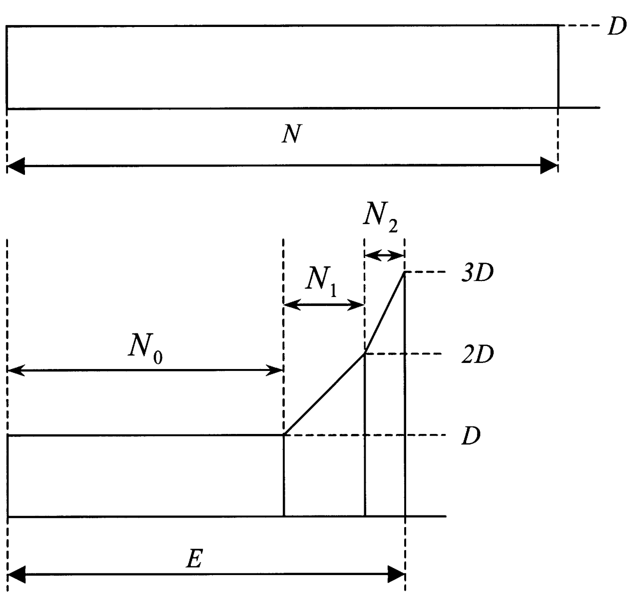

Figure 1. Measurement of single particle diameters vs. cluster length in identical–diameter case.

Definitions

Cluster Order in the Identical–Diameter Case

The order of a cluster depends on its total length. The cluster lengths were classified into intervals based on the identical diameter (D) of the particles within the cluster (this process is referred to as “thresholding”). A “single” is a cluster with a length in the range [0,D], a “double” is a cluster with a length in the range [D,2D], etc.

Singles, doubles, triples, and quads are referred to as clusters of order 1, 2, 3, and 4, respectively. The number of clusters per order is indicated by the symbols N_0 (singles), N_1 (doubles), and N_2 (triples), as shown in figure 1. Note that the total number of cluster detections (E) is equal to $N_0 + N_1 + N_2$, etc.

An n–particle cluster was defined as a cluster containing n particles. For example, depending on the cluster length, a 4–particle cluster can be a double, a triple, or a quad.

Cluster Order in Distributed–Diameter Cases

In the case of distributed diameters, the definition of cluster orders becomes quite obscure. In the identical–diameter case, a single is unique (no cluster can be smaller than D). However, in distributed–diameter cases, a single has to be related to the mean diameter of the particles. The length of a single can indeed be a single particle of mean diameter, but it can also be a cluster composed of two smaller overlapping particles. Nonetheless, the same principle was used for the definition of the cluster orders, in this case based on the mean (μ) and standard deviation (σ) of the particles. Singles were defined as clusters with lengths in the interval [0,μ + 3σ], doubles as clusters with lengths in the interval [μ + 3σ,2μ + 3σ], triples as clusters with lengths in the interval [2μ + 3σ,3μ + 3σ], etc.

Occupancy Rate (*OR*)

The occupancy rate is a dimensionless variable that acts as a measure of the flow density. It was defined as the total length of all particles in an experiment, divided by the total “space” passing in a certain time interval. In practice, this space is obtained by multiplying the mean velocity of all clusters by the time duration of the experiment:

$$OR = \frac{N\bar{D}}{\bar{v}t} \qquad (1)$$

where

N = number of particles in experiment

$\bar{D}$ = mean diameter of particles (m)

$\bar{v}$ = mean velocity of the clusters (ms^{-1})

t = total duration of the experiment (s)

In the single–particle approach (SPA), this parameter tends to “0” because the probability of having no overlaps can only be present when the flow is very sparse.

Event Ratio (*ER*)

The number of cluster detection events (E) divided by the number of particles used in the simulation was defined as the event ratio (ER):

$$ER = \frac{E}{N} \qquad (2)$$

In the single–particle approach (SPA), this parameter tends to “1” because each individual particle produces an event.

Simulation Procedure

The algorithm used for simulations was written in MatLab (1997). The procedure for cluster detection was as follows:

A tail (T) location vector was defined, containing numbers in an interval dictated by the occupancy rate (OR) and the number of particles. For example, suppose there are 5000 particles with a mean diameter of 5 mm in the simulation, and OR = 0.5. The total length of the particles would be 5 × 5000 = 25000 mm, and the total "space" available should be 25000/(0.5) = 50000. In this case, the T locations are uniformly distributed in the interval [0,50000].

Diameters (D) were assigned to each T location. The diameter value can be a constant (for identical particles), or drawn from a given distribution (uniform or normal).

The tail (T) location vector was sorted, keeping the assigned diameters with the original locations. The tail locations must be incremental to ensure proper functioning of the detection algorithm.

Cluster detection is based on a simple comparison between the head of the current cluster and the tail of the next particle. First, T_1 is assigned to the cluster tail, as shown in figure 2.

The current head (H_1) is compared with the next particle tail (T_2). If $H_1 > T_2$, then there is overlap. The new cluster head ($\overline{H_2}$) is now the maximum of H_1 and H_2 since it is possible that $H_1 > H_2$ in distributed–diameter cases. The same process is used to detect that $\overline{H_2} > T_3$, and H_3 becomes the new cluster head ($\overline{H_3}$).

The next comparison ($\overline{H_3} > T_4$) is false in the illustration shown in figure 2. This gap marks the occurrence of a new cluster.

The current cluster length (C_1) is now equal to the current cluster head ($\overline{H_3}$)minus the current cluster tail location (T_1). Note that it is not necessary to check the next particle tail (T_5) for possible overlap. Because the tail locations were incrementally sorted, it is impossible for a higher tail to overlap if the lower tail does not.

This procedure was repeated until all particles had been considered.

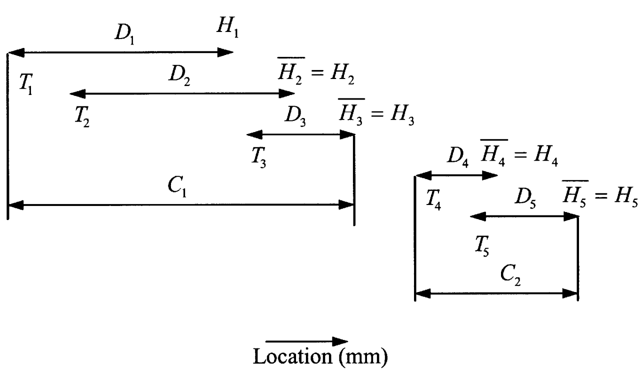

Figure 2. Cluster detection during simulation.

Exponential Estimator Hypotheses

The Exponential Estimator algorithm evolved from validation of two intuitive hypotheses:

1. The number of detected clusters is related to the flow density: higher density means fewer clusters, and fewer clusters means more particles per cluster:

$$ER = \frac{E}{N} = f(OR) \quad (3)$$

2. The ratio of cluster orders — for example, the ratio of doubles–singles ($i = 0$) — is also related to the flow density.

The ratio of cluster orders is a measurable quantity:

$$OR = g\left(\frac{N_{i+1}}{N_i}\right) \quad (4)$$

Using equations 3 and 4, the original number of particles (N) can now be estimated:

$$N_{EST} = Ef^{-1}(OR) = Ef^{-1}\left[g\left(\frac{N_{i+1}}{N_i}\right)\right] \quad (5)$$

In this formula, the number of events (E), as well as the number of clusters per order (N_{i+1},N_i), can be measured. Simulations were carried out to determine the functional relationships f and g.

Results

Simulations were carried out for four different particle–diameter cases:

- I: Identical diameters (5 mm)
- II: Uniformly distributed diameters (4–6 mm)
- III: Normally distributed diameters (mean = 5 mm, s.d. = 1 mm)
- IV: Urea–distributed diameters (mean = 2.58 mm, s.d. = 0.41 mm)

All simulations were carried out with 5000 initial particles (N in fig. 1). In the figures 3 through 6, the data is organized as follows: In the top–left subplot (a), the sorted cluster lengths are represented for an occupancy rate (OR) of 0.5. The top–right subplot (b) represents the relationship between OR (in the range [0,1]) and the event ratio (ER):

$$ER = \frac{E}{N} = f(OR) \quad (6)$$

This relationship was found to be exponential and identical for all four particle–diameter cases. In the bottom–left subplot (c), the relationship between the ratio of doubles–singles and OR is shown:

$$OR = g\left(\frac{N_{i+1}}{N_i}\right) \quad (7)$$

It was found that the ratio of doubles–singles sufficed to perform the reconstruction (i = 0). This relationship was found to be straight–line in all four particle–diameter cases. In the identical–diameter case (I), the slope was found to be unity. In the distributed–diameter cases (II, III, and IV), it was found to be less than unity. The bottom–right subplot (d) is the combination of the previous two functional relationships:

$$N_{EST} = Ef^{-1}(OR) = Ef^{-1}\left[g\left(\frac{N_{i+1}}{N_i}\right)\right] \quad (8)$$

This plot represents the complete Exponential Estimator. In all four particle–diameter cases, the complete Exponential Estimator is, as the name implies, exponential. In the identical–diameter case (I), there is no coefficient necessary in the power (it is unity). In the distributed–diameter cases (II, III, and IV), coefficients are necessary.

Case I: Identical–Diameter Particles

In the simulation of identical–diameter particles (5 mm), singles, doubles, and triples were defined as clusters with lengths in the intervals [0,5] mm, [5,10] mm, and [10,15] mm, respectively. The sorted cluster lengths that resulted under these conditions are shown in figure 3a.

Figure 3a reveals a shape in which a significant number of clusters are singles (interval [0, $\approx$1900]), there is a linear increase in length for doubles (interval [$\approx$1900, $\approx$2800]), but the increase for triples (interval [$\approx$2800, $\approx$3000]) is not linear. In fact, it seems more quadratic. Note also that clusters of order 4 and 5 occur, although their numbers are small.

To test the hypotheses, simulations were carried out for various occupancy rates (*OR*, see eq. 1) in the range [0,1]. The simulations revealed a remarkably simple relationship between *OR* and the event ratio (*ER*, see eq. 2):

$$ER = e^{-OR} \tag{9}$$

This relationship was found by plotting the occupancy rate against the logarithm of the event ratio and fitting a straight line in an Ordinary Least Squares sense, as shown in figure 3b. The slope of the regression line is –1.00.

The origin was regarded as a fixed point of the regression line. This is justified since the event ratio tending to “1” (and its logarithm tending to “0”) implies the single–particle approach (SPA). In this case, all particles are measured individually, and the occupancy rate (which represents the flow density) must tend to “0.” Combination of equation 2 and equation 9 gives:

$$\frac{E}{N} = e^{-OR} \tag{10}$$

As an example, the predicted number of events for a given *OR* of 0.5 would be:

$$E = Ne^{-OR} = 5000 \times e^{-0.5} = 3033 \tag{11}$$

which is suggested by figure 3a, in which the total number of clusters was indeed approximately 3000, out of 5000 initial particles. Since the number of events is known from measurements, the total number of particles can be estimated by rewriting equation 10 as:

$$N_{EST} = E \times e^{OR} \tag{12}$$

This method seems impractical because the occupancy rate is not measured directly. However, for identical diameters, another simple relationship was found between the ratio of doubles–singles (which can be measured) and the occupancy rate:

$$OR = \frac{N_1}{N_0} \tag{13}$$

This relationship is shown in figure 3c. The slope of the regression line is 0.99. Equation 13 can be also be verified from figure 3a, which suggests twice as many singles as doubles (*OR* = 0.5). Combination of equation 12 and equation 13 yielded the complete Exponential Estimator for identical particles based on doubles and singles:

$$N_{EST1\_0} = E \times e^{N_1/N_0} \tag{14}$$

The complete Exponential Estimator was obtained by plotting the ratio of doubles–singles against the logarithm of the event ratio, as shown in figure 3d. The slope of the regression line is –1.01, again using the origin as a fixed point.

Case II: Uniformly Distributed Diameter Particles

The sorted cluster lengths of particles with uniformly distributed diameters ([4,6] mm) are shown in figure 4a. As in figure 3a, approximately 3000 clusters emerged from 5000 initial particles, which suggests that equation 9 ($ER = e^{-OR}$) is valid for uniformly distributed diameters as well as identical diameters. For singles (determined from threshold-ing), a gradual increase is apparent. However, doubles are not quite as abundant as in the identical diameter case, which suggests that equation 13 ($OR = N_0/N_1$) is no longer valid.

In figure 4b, the occupancy rate was plotted against the logarithm of the event ratio. A straight line with the origin as fixed point was fitted to verify the relationship, as was obtained from the identical–diameter data. The slope of the regression line is –1.00, which indicates that equation 9 is also correct in the uniformly distributed diameter case.

Figure 4c plots the ratio of doubles–singles against the occupancy rate. The functional relationship seems a straight line, but the slope is no longer unity, as was the case for identical diameters. Therefore, for particles of uniformly distributed diameters, the relationship was modeled as:

$$OR = \alpha \frac{N_1}{N_0} \tag{15}$$

where α is a material–specific parameter. The slope of the straight line is –0.63, leading to a value for α of 1.59.

The complete Exponential Estimator for particles of uniformly distributed diameters was obtained using the slope (–0.63) of the regression line in figure 4d:

$$N_{EST1\_0} = E \times e^{1.59\left(N_1/N_0\right)} \tag{16}$$

Case III: Normally Distributed Diameter Particles

In this case, a normal distribution of diameters was used with a mean (μ) of 5 mm and a standard deviation (σ) of 1 mm. In figure 5a, the sorted cluster lengths are shown for an occupancy rate of 0.5. As was the case for identical diameters and uniformly distributed diameters (figs. 3a and 4a), approximately 3000 events occurred from 5000 initial particles. The relationship between the occupancy rate and the logarithm of the event ratio is shown in figure 5b. The slope of the regression line is –1.00.

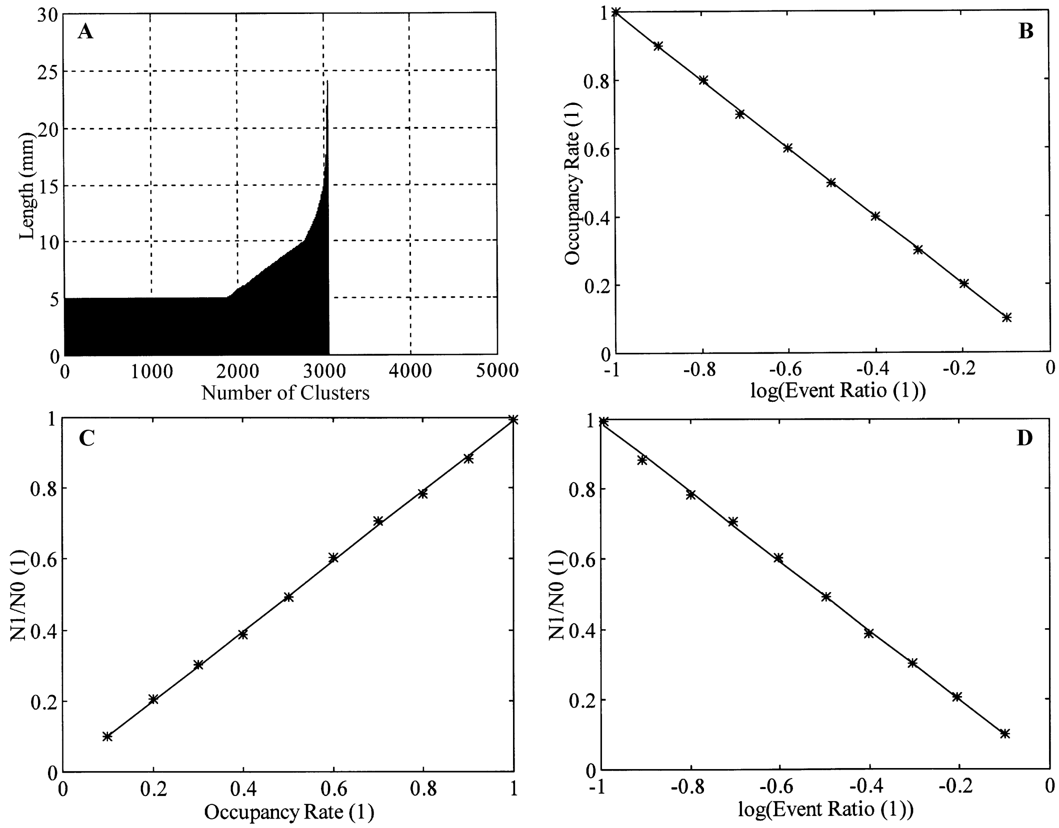

Figure 3. Sorted cluster lengths (*OR* = 0.5, 5000 initial particles) and functional relationships for identical particles (5 mm).

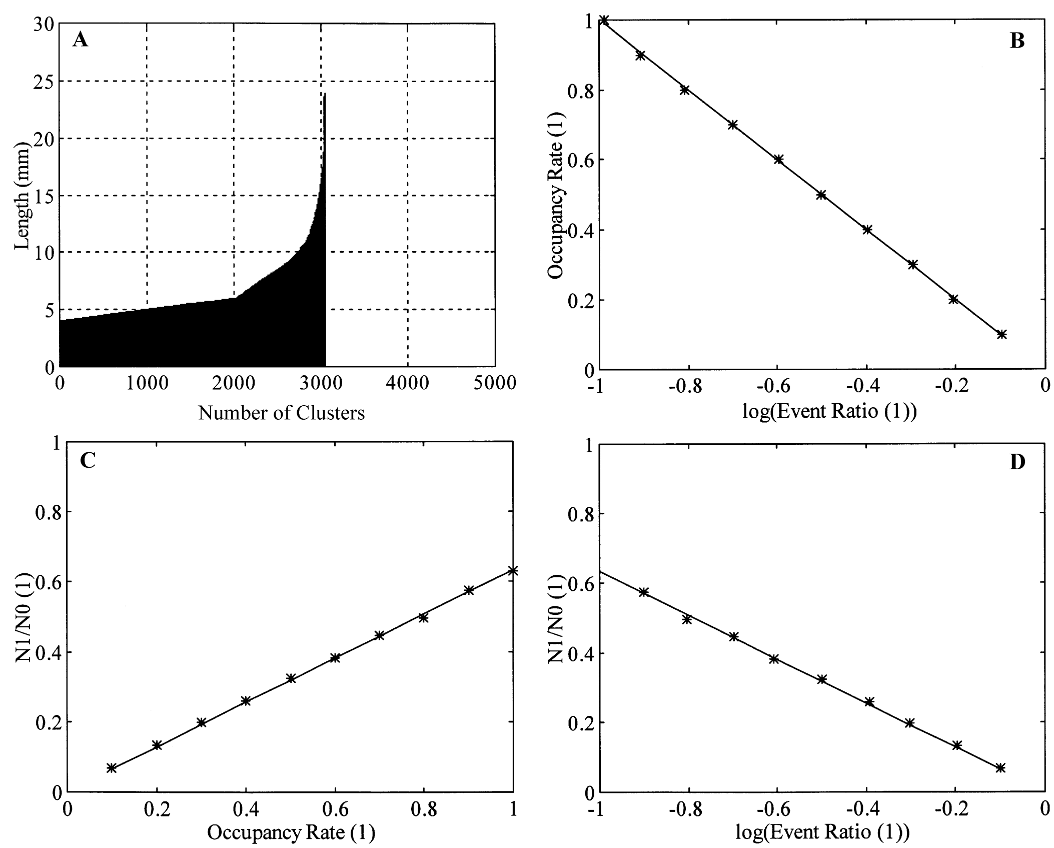

Figure 4. Sorted cluster lengths (*OR* = 0.5, 5000 initial particles) and functional relationships for uniformly distributed particles (4–6 mm).

As with uniformly distributed diameters (case II), the relationship expressed by equation 9 was found to be valid for normally distributed diameters as well. It does not depend on the diameter distribution, nor on the mean of the diameters. However, as was the case for uniformly distributed diameters, the ratio of doubles–singles (again determined by thresholding) depends on the diameter variability, as indicated by figure 5c. The slope of the regression line is 0.43.

The complete estimator follows from the slope (–0.43) of the regression line in figure 5d (α = 2.33). The Exponential Estimator for particles of normally distributed diameters (μ = 5 mm, σ = 1 mm) is:

$$N_{EST1\_0} = E \times e^{2.33(N_1/N_0)} \tag{17}$$

CASE IV: UREA–DISTRIBUTED DIAMETER PARTICLES

Simulations were carried out for diameters obtained from urea fertilizer to compare with measured data. The distribution was determined by dropping 600 particles through an optical sensor, as described in Part 2 of this research (Grift et al., 2001). The distribution had a mean (μ) of 2.58 mm and a standard deviation (σ) of 0.41 mm. It was also skewed and, hence, non–Gaussian.

In figure 6a, the cluster lengths of urea particles are shown for an occupancy rate of 0.5. As is clear from the plot, approximately 3000 clusters emerged from 5000 initial particles. As in the three other cases, the relationship expressed by equation 9 proved to be valid for a skewed distribution. This is confirmed by figure 6b, which shows a linear regression line with slope of –1.00. Figure 6c plots the ratio of doubles–singles against the occupancy rate. The regression line is linear, and the slope is 0.55.

The complete estimator follows from the slope (–0.55) of the regression line in figure 6d (α = 1.8). The Exponential Estimator for urea–distributed particles (μ = 2.58 mm, σ = 0.41 mm) is:

$$N_{EST1\_0} = E \times e^{1.8(N_1/N_0)} \tag{18}$$

The functional relationships and the values of parameter α for the four different particle–diameter cases are shown in table 1.

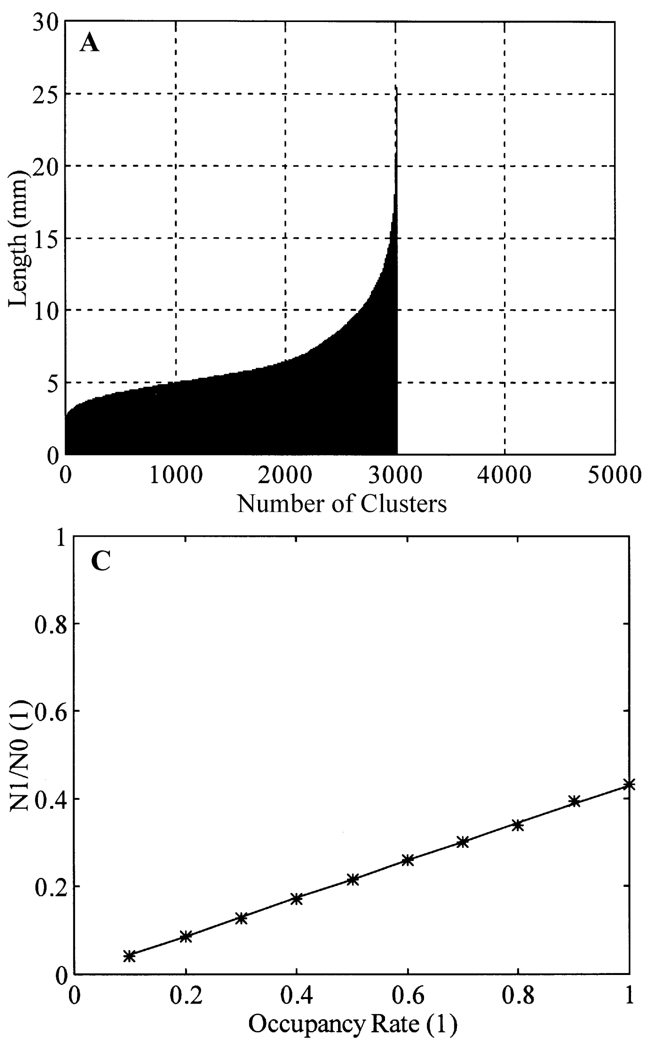

Figure 5. Sorted cluster lengths (*OR* = 0.5, 5000 initial particles) and functional relationships for normally distributed particles (mean = 5 mm, s.d. = 1 mm).

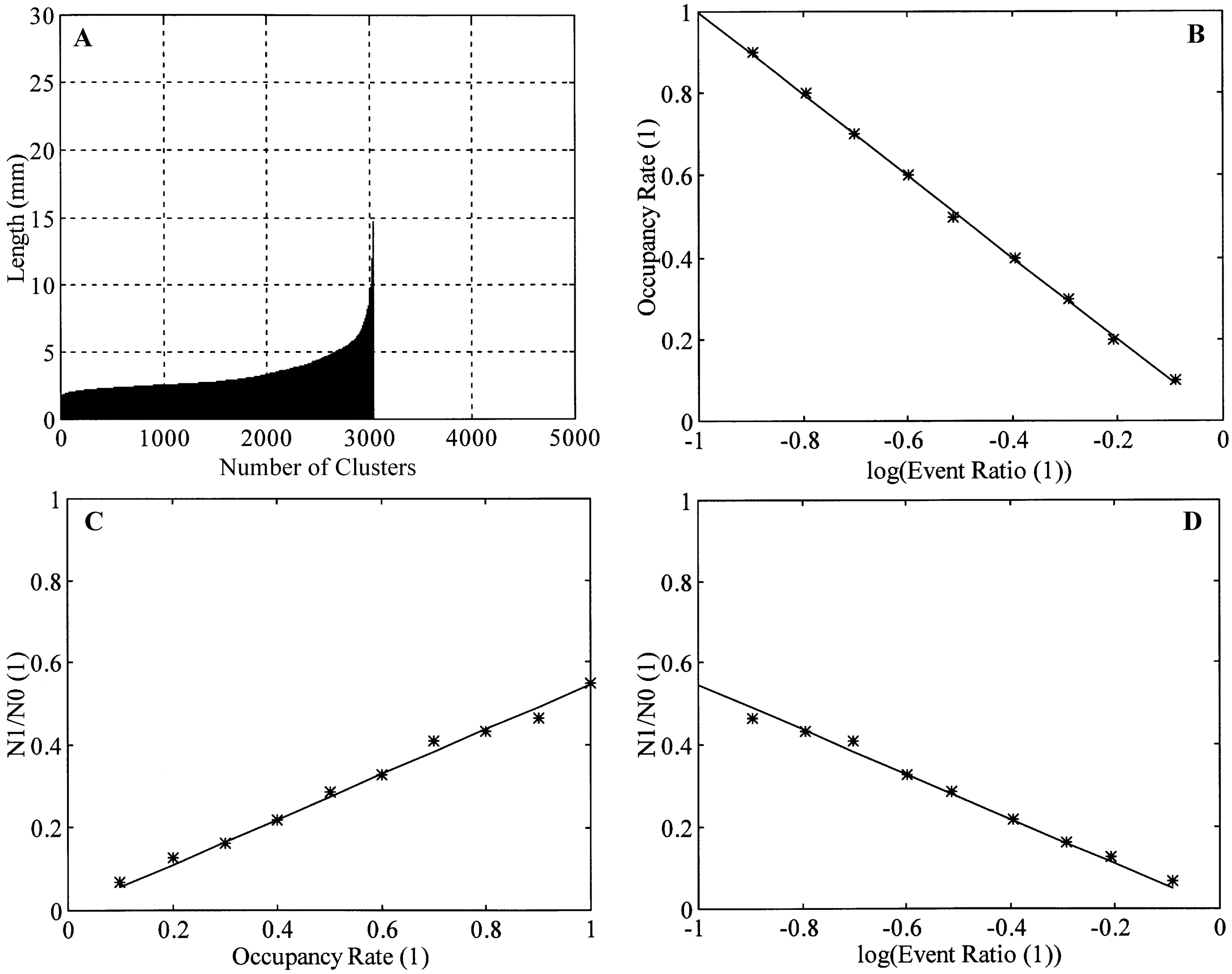

Figure 6. Sorted cluster lengths (*OR* = 0.5, 5000 initial particles) and functional relationships for urea–distributed particles (mean = 2.58 mm, s.d. = 0.41 mm).

Table 1. Functional relationships of the Exponential Estimator for the four particle–diameter cases.

	Case I: Identical–Diameter Particles (5 mm)	Case II: Uniformly Distributed Diameter Particles ([4,6] mm)	Case III: Normally Distributed Diameter Particles (µ= 5 mm, ó = 1 mm)	Case IV: Urea–Distributed Diameter Particles (µ = 2.58 mm, ó = 0.41 mm)
$ER=\frac{E}{N}=f(OR)$	$ER=e^{-OR}$	$ER=e^{-OR}$	$ER=e^{-OR}$	$ER=e^{-OR}$
$OR=g\left(\frac{N_{i+1}}{N_i}\right)$	$OR=\frac{N_1}{N_0}$	$OR=\alpha\frac{N_1}{N_0}$	$OR=\alpha\frac{N_1}{N_0}$	$OR=\alpha\frac{N_1}{N_0}$
$N_{EST}=Ef^{-1}\left[g\left(\frac{N_{i+1}}{N_i}\right)\right]$	$N_{EST1\_0}=E\times e^{N_1/N_0}$	$N_{EST1\_0}=E\times e^{\alpha(N_1/N_0)}$ $\alpha=1.59$	$N_{ESTI\_0}=E\times e^{\alpha(N_1/N_0)}$ $\alpha=2.32$	$N_{ESTI\_0}=E\times e^{\alpha(N_1/N_0)}$ $\alpha=1.8$

CONCLUSIONS

A method called the "Exponential Estimator" was obtained through simulation of clustering processes. This method allows the measurement of fertilizer mass flow in aerial spreader ducts under high flow densities.

A basic law was found that estimated the original number of particles in the flow from the total number of clusters measured in a certain time period:

$$N_{EST} = E \times e^{OR} \tag{19}$$

where N_{EST} is the estimated number of particles, E is the number of clusters detected, and OR is the occupancy rate, a measure of the flow density. This relationship was found to be universal, and independent of the diameter distribution and the mean diameter. The occupancy rate is not directly measured, which seemingly limits the applicability of the equation. However, for identical–diameter particles, the occupancy rate was found to have a remarkably simple relationship with the measurable ratio of doubles–singles (the number of clusters with lengths in predefined categories):

$$OR = \frac{N_1}{N_0} \tag{20}$$

which leads to the Exponential Estimator for identical–diameter particles:

$$N_{EST1\_0} = E \times e^{N_1/N_0} \tag{21}$$

For distributed–diameter particles, equation 20 did not apply. The relationship between the occupancy rate and the ratio of doubles–singles was still linear, but the slope was no longer unity. For these cases, a material–specific parameter (α) was introduced which depended solely on the diameter variability, not on the mean diameter. The Exponential Estimator for distributed–diameter materials is:

$$N_{EST1\_0} = E \times e^{\alpha(N_1/N_0)} \tag{22}$$

The Exponential Estimator is a simple reconstruction method. It requires only the determination of the number of singles and doubles from measurements and the total number of cluster passages in a time frame. More in–depth research on the material–specific parameter (α) could reveal a fundamental relationship with the moments of its diameter distribution.

The simplicity of the relationships found suggests the possibility of deriving the Exponential Estimator using a theoretical approach. The simulation tool as developed here could be a valuable instrument for this purpose.

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