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Research Paper: PM—Power and Machinery

Estimating mean particle diameter in free-fall granular particle flow using a Poisson model in space

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ARTICLE INFO

Article history:
Received 28 August 2007
Received in revised form
19 April 2008
Accepted 11 June 2008
Available online 27 July 2008

The mean diameters of particles in a free-falling flow regime were estimated using a combination of theory and measurements. The flow regime consisted of an intermittent sequence of clumps of particles separated by spacings among them. The objective of this research was to determine the mean diameters of particles in the flow, based on measurement of the lengths of the clumps and spacings. The model used to calculate the diameter estimate was based on the assumption that the flow forms a Poisson process, where particles arrive at a time-of-flight sensor independently in space. This assumption implies that the flow density in particles per metre can be obtained by taking the reciprocal value of the mean length of the spacings among clumps of particles. This facilitated the derivation of a method for estimating the theoretical mean particle diameter.

Discrepancies between the measured data and its theoretical counterpart could be caused by (1) the Poisson assumption not being valid and (2) measurement errors. To isolate these effects the Poisson assumption was tested. The result was that among 40 datasets, grouped into 10 replicates taken at four different fall heights causing four different flow densities, 30 were assumed to be Poisson driven. The 10 remaining datasets had a high density at which point the Poisson assumption appeared to break down. Subsequently, using only the 30 experiments which were assumed Poisson driven, the sensor was characterised by equalising predicted clump and spacing lengths with their measured equivalents. This yielded two sensor characteristic constants, which were used in the measurement model to estimate the mean diameter of the particles among a range of flow densities.

The diameter of 4.5 mm particles was estimated using the 30 datasets assumed to form a Poisson process, with a mean value of 4.50 mm and a standard deviation of 0.044 mm (1% coefficient of variation) in a flow density range from 67 particles m^{-1} to 167 particles m^{-1}. The maximum and minimum values were 4.57 mm (+1.6% error) and 4.42 mm (−1.8%), respectively.

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1. Introduction

The flowrate of granular materials is an important measure for monitoring and control in agricultural applications such as in fertiliser application, the transport of seeds and fertiliser in the tubes of pneumatic planters, as well as in flows of grains and fruits during harvesting (yield monitoring). Grift and others (Grift, 2001; Grift *et al.*, 2001; Grift, 2003) devised methods to measure the flowrate by measuring the lengths of clumps of particles and spacings among them. However,

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1537-5110/$ – see front matter Published by Elsevier Ltd on behalf of IAgrE.
doi:10.1016/j.biosystemseng.2008.06.005

Nomenclature

N	initial number of particles in experiment
N_T	total number of clump/spacing arrivals

Theoretical quantities

$\overline{D}_{Theor}$	theoretical mean diameter of particles, m
$CL_{i,Theor}$	theoretical length of ith clump, m
$SL_{i,Theor}$	theoretical length of ith clump, m
$\overline{CL}_{Theor}$	mean of theoretical clump lengths over N_T measurements, m
$\overline{SL}_{Theor}$	mean of theoretical spacing lengths over N_T measurements, m
λ_{Theor}	theoretical flow density, m^{-1}
TL_{Theor}	theoretical total length passing during experiment, m

Measured quantities

$\hat{\overline{D}}$	estimator for mean diameter of particles, m
CL_i	length of ith clump, m
SL_i	length of ith spacing, m
$\overline{CL}$	mean of clump lengths over N_T measurements, m
$\overline{SL}$	mean of spacing lengths over N_T measurements, m
TC_i	interruption time of ith clump, s
TS_i	interruption time of ith spacing, s
TF_i	flank time representing the velocity of ith clump and spacing
v_i	velocity of ith clump/spacing, ms^{-1}

Characterisation quantities

b_S	characteristic value by which sensor measures spacing lengths, m
b_C	characteristic value by which sensor measures clump lengths, m
$\hat{b}_S$	estimator for mean characteristic value by which sensor measures spacing lengths, m
$\hat{b}_C$	estimator for characteristic value by which sensor measures clump lengths, m

in these methods the mean particle diameter was obtained using off-line measurements rather than from the on-line data. The mean particle diameter can be used for purposes such as on-line mass flow measurements, where the flowrate in particles per second is multiplied by the mean particle volume and material density. Secondly, the mean particle diameter can be used as an input to ballistic models, for instance to predict landing locations of fertiliser particles in aerial application as proposed by Grift and Hofstee (2002).

Estimation of the flowrate, expressed in particles per time unit, was shown possible by assuming a Poisson arrival process *in time*, which requires merely a single plane photo-interruption device since no information about lengths of either clumps or spacings is needed (Grift and Crespi, 2008). In contrast, to measure the mean particle diameter as discussed here, a dual layer photo-interruption device is needed, commonly termed a time-of-flight device. Here the assumption was made that the particles arrive at the sensor according to a Poisson arrival process *in space*. In other words, it is assumed that the particles arrive at the sensor independently, in a similar way to classical examples from queuing theory such as telephone calls arriving at a helpdesk. If the process is indeed Poisson driven, the flowrate equivalent 'density', expressed here in particles per length unit, is equal to the reciprocal value of the mean of the spacing lengths (Hall, 1988). This assumption is essential to derive a method of estimating the theoretical mean particle diameter. In practice, it is not necessary to have all the inter-particle lengths which cannot be measured due to the clumping effect. Since only the mean value of the spacing lengths is required, the sum of the inter-clump lengths divided by the number of clumps (which is counted without error) can be used.

Incorporating the spacing lengths yields a major simplification compared to earlier attempts which focused on solely the clump lengths (Grift, 2003), since the theoretical distribution of the clump lengths is complicated (Daley, 2001; Crespi and Lange, 2006). The intermittent sequence of spacings and clumps formed by a Poisson process is often termed as a coverage process or simple linear Boolean model (Hall, 1988). Other applications of the simple linear Boolean model are found in particle and electronic counters (Takacs, 1962), diffusion of suspended particles (Bingham and Dunham, 1997), Markov/General/∞ queue (Kleinrock, 1975) and biomedical applications (Crespi *et al.*, 2005).

The objective of this research was to estimate the mean particle diameter of free-falling granular particles using only measurements of clump and spacing lengths.

2. Theoretical framework

In this section, a theoretical method for estimating the mean particle diameter assuming a Poisson driven granular particle flow is derived. Fig. 1 shows a simplified granular flow regime where particles form clumps with spacings among them. Grift (2001) defined the occupancy rate OR_{Theor}, a measure of flow density as follows:

$$OR_{Theor} = \frac{N\overline{D}_{Theor}}{TL_{Theor}} \tag{1}$$

where N is the initial number of particles in the experiment, $\overline{D}_{Theor}$ is the mean particle diameter in m, and TL_{Theor} in m is the total length of the space passing during the experiment. Grift (2001) also stated that the initial number of particles N is related to the occupancy rate as follows:

$$N = N_T e^{OR_{Theor}} \tag{2}$$

where N_T is the number of clump/spacing pairs counted during the experiment. In Fig. 1 for example, among $N = 20$ initial particles $N_T = 8$ clump/spacing pairs are formed and the occupancy rate is equal to the natural logarithm of 20/8 being 0.916.

The flow density λ_{Theor} in particle m^{-1} is equal to the number of particles in the experiment N divided by the total length passing during the experiment TL_{Theor} in m, or

$$\lambda_{Theor} = \frac{N}{TL_{Theor}} \tag{3}$$

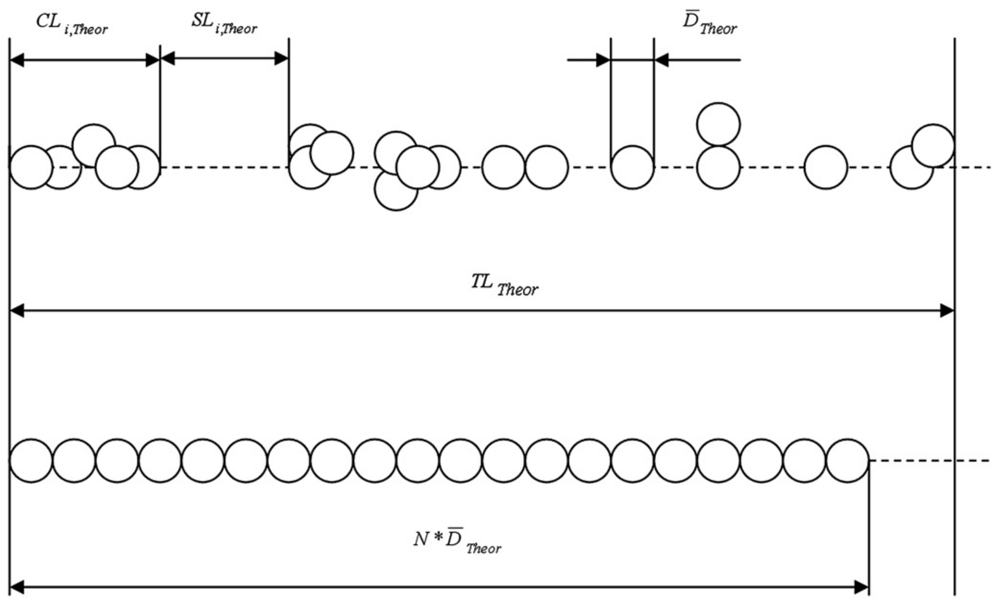

Fig. 1 – Granular flow of particles forming clumps and spacings.

By substitution of Eq. (3) into Eq. (1) the relationship between the occupancy rate OR_{Theor} and the flow density λ_{Theor} in m^{-1} is obtained:

$$OR_{Theor} = \lambda_{Theor}\bar{D}_{Theor} \quad (4)$$

Substituting the occupancy rate OR_{Theor} from Eq. (2), into Eq. (4) the mean particle diameter can be expressed as

$$\bar{D}_{Theor} = \frac{OR_{Theor}}{\lambda_{Theor}} = \frac{1}{\lambda_{Theor}}\ln\left(\frac{N}{N_T}\right) \quad (5)$$

This equation for the mean diameter contains the initial number of particles (which is not known in practice) and the flow density λ_{Theor}, which is not measured directly. However, when assuming a stationary Poisson process in space, the flow density λ_{Theor} in m^{-1} is equal to reciprocal value of the mean spacing lengths $\overline{SL}_{Theor}$ in m (Hall, 1988) or

$$\lambda_{Theor} = \frac{1}{\overline{SL}_{Theor}} \quad (6)$$

Therefore the mean diameter can be written after substitution of Eq. (6) into Eq. (5):

$$\bar{D}_{Theor} = \overline{SL}_{Theor}\ln\left(\frac{N}{N_T}\right) \quad (7)$$

This step replaced the flow density by the measurable mean spacing lengths, but the unknown initial number of particles N remains to be replaced by a quantity measurable in practice. The total length of space TL_{Theor} in m, passing during the experiment is equal to the sum of the clump and spacing lengths. This can be written in terms of the means of the clump and spacing lengths of which N_T are counted during an experiment:

$$TL_{Theor} = \sum_{i=1}^{N_T}(CL_{i,Theor} + SL_{i,Theor}) = N_T(\overline{CL}_{Theor} + \overline{SL}_{Theor}) \quad (8)$$

The total number of particles in the experiment is equal to the flow density λ_{Theor} in m^{-1} from Eq. (6) multiplied by the total length TL_{Theor} in m from Eq. (8) or

$$N = \lambda_{Theor} \times TL_{Theor} = \frac{1}{\overline{SL}_{Theor}}N_T(\overline{CL}_{Theor} + \overline{SL}_{Theor}) = N_T\left(\frac{\overline{CL}_{Theor}}{\overline{SL}_{Theor}} + 1\right) \quad (9)$$

This implies that the ratio (N/N_T) in Eq. (5) can be written as a function of measurable quantities being the mean clump lengths $\overline{CL}_{Theor}$ in m and mean spacing lengths $\overline{SL}_{Theor}$ in m:

$$\frac{N}{N_T} = \left(\frac{\overline{CL}_{Theor}}{\overline{SL}_{Theor}} + 1\right) \quad (10)$$

Thus, an equation for the mean particle diameter $\hat{\bar{D}}_{Theor}$ in m can be written in terms of quantities $\overline{CL}_{Theor}, \overline{SL}_{Theor}$ (which are values that can be measured in practice) by substitution of Eq. (6) and Eq. (10) into Eq. (5):

$$\hat{\bar{D}}_{Theor} = \overline{SL}_{Theor}\ln\left(\frac{\overline{CL}_{Theor}}{\overline{SL}_{Theor}} + 1\right) \quad (11)$$

2.1. *Simulation example of diameter estimation in a Poisson driven flow experiment*

To illustrate the calculation method, a simulation of a Poisson flow experiment is shown in Fig. 2. Here 4000 particles were used with Gaussian distributed diameters, a mean of 4.5 mm and a standard deviation of 0.5 mm. The flow density was set to 100 particles m^{-1}, which resulted in 2547 clump/spacing pairs. The resulting mean clump and spacing lengths were 5.6901 mm and 9.957 mm, respectively. The

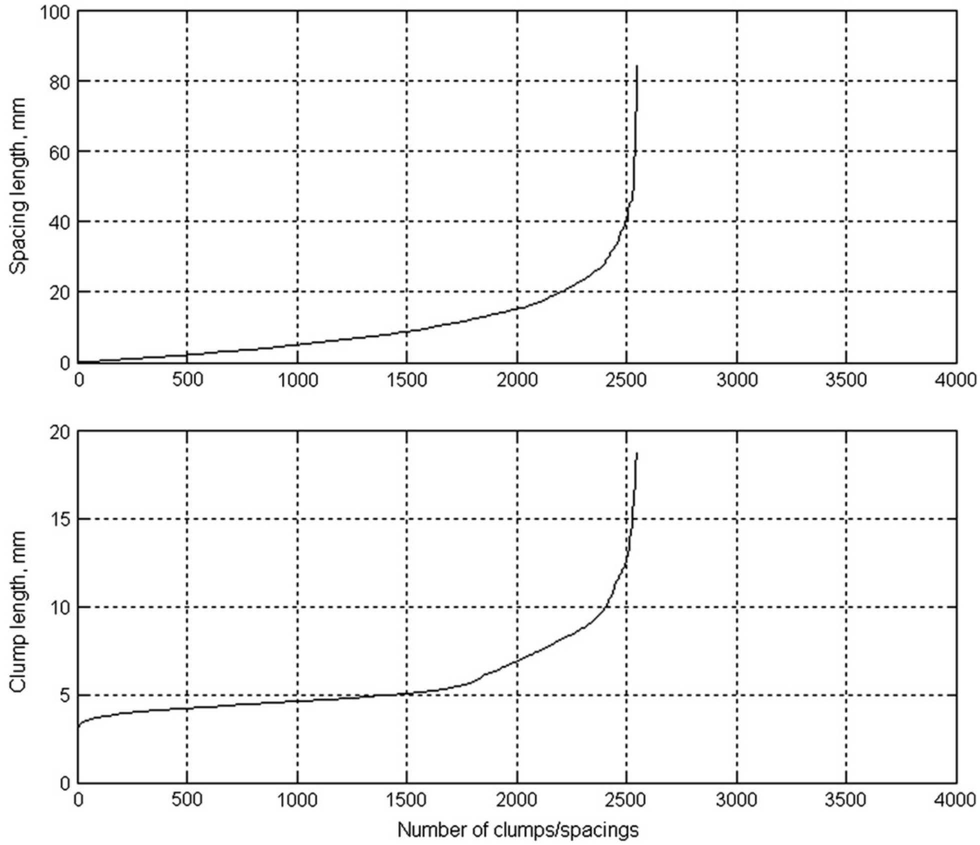

Fig. 2 – Simulated Poisson process where 4000 particles with a Gaussian distributed diameter with a mean of 4.5 mm and standard deviation of 0.5 mm were used at a flow density of 100 particles m^{-1}.

mean particle diameter can now be estimated using Eq. (11) as follows:

$$\widehat{\overline{D}}_{\text{Theor}} = \overline{\text{SL}}_{\text{Theor}} \ln\left(\frac{\overline{\text{CL}}_{\text{Theor}}}{\overline{\text{SL}}_{\text{Theor}}} + 1\right) = 9.957\ \text{mm} \times \ln\left(\frac{5.6901\ \text{mm}}{9.957\ \text{mm}} + 1\right) = 4.5007\ \text{mm}$$

The flow density can also be estimated, using Eq. (6): $\lambda_{\text{Theor}} = 1/\overline{\text{SL}}_{\text{Theor}} = 1/9.957\ \text{mm} = 100.4$ particles m^{-1}.

2.2. *Sensor characterisation*

Estimating the mean diameter using Eq. (11) is based on a theoretical concept, which relies on the principle that the mean clump and spacing lengths are measured without error. In practice the sensor measures both the length of the clumps and spacings with a certain error. The sensor is an optical device that projects magnified images of passing clumps onto two sets of receiver arrays as shown in Fig 3. The sensor was adjusted such that particles falling in the centre of the tube are projected in focus, but particles falling away from the centre are observed out of focus. In addition, the exact switching points of the optical sensor arrays are unknown, since they depend on the intensity of the light they receive. A third error is caused by the fact that each sensor array contains 30 discrete receivers, and particles may either fall in-line with the receivers or offset from their centrelines.

To accommodate for the errors in the measured clump and spacing lengths the system was characterised. Fig. 4 shows an idealised interruption process. The velocity of the ith clump v_i in m s^{-1} is equal to the distance between the sensor arrays b in m, divided by the time TF_i in s required to move the clump head from the upper to the lower interruption plane, or from event (1) to event (2) or

$$v_i = \frac{b}{\text{TF}_i} \tag{12}$$

where a constant velocity was assumed since the distance between the interruption planes b is small (approximately 1 mm). The length of the clump CL_i in m can now be measured by multiplying the clump velocity v_i by the time interval TC_i in s during which the clump interrupts either plane from (1) to (3) or (2) to (4):

$$\text{CL}_i = v_i \text{TC}_i = \frac{b}{\text{TF}_i}\text{TC}_i = b\frac{\text{TC}_i}{\text{TF}_i} \tag{13}$$

In reality the clump lengths are measured with a certain error as mentioned. Therefore, instead of using a single parameter b in m representing the distance between the interruption planes, a lumped parameter b_C in m was introduced which is equal to the distance b multiplied by an error compensation factor.

$$\text{CL}_i = v_i \text{TC}_i = b_C\frac{\text{TC}_i}{\text{TF}_i} \tag{14}$$

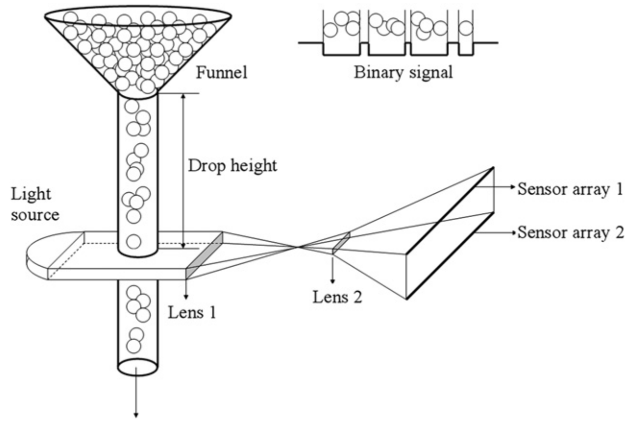

Fig. 3 – Experimental arrangement in which particles are dropped from a funnel through an optical time-of-flight sensor. The sensor measures the velocity of clumps using the time difference between the interruption of sensor array 1 to sensor array 2. The lengths of clumps and spacings are measured using the time during which the clump interrupts either sensor array in combination with the measured velocity. The lenses magnify the image of the clumps onto the sensor arrays, which allows using relatively large sensors (5 mm) to detect particles as small as 1 mm.

The lumped parameter b_C was subsequently characterised using measurement data. The mean value of the clump lengths is now

$$\overline{CL} = \frac{b_C}{N_T}\sum_{i=1}^{N_T}\frac{TC_i}{TF_i} \tag{15}$$

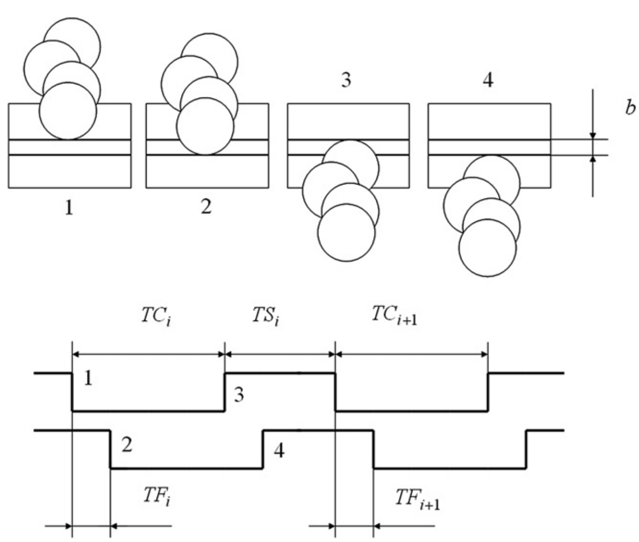

Fig. 4 – Time-of-flight sensor, which records interruption times to measure clump and spacing lengths. The velocity of the clump corresponds to the clump travelling through a distance *b*, (1) to (2), and the length of the clump follows from multiplying the velocity by the time during which the clump passes one sensing line representing a sensor array (from (1) to (3)).

Solving for b_C this gives

$$b_C = \overline{CL}\frac{N_T}{\sum_{i=1}^{N_T}\frac{TC_i}{TF_i}} \tag{16}$$

To calculate the value of b_C, the measured mean clump length $\overline{CL}$ in m was replaced by its theoretical equivalent:

$$\hat{b}_C = \overline{CL}_{Theor}\frac{N_T}{\sum_{i=1}^{N_T}\frac{TC_i}{TF_i}} \tag{17}$$

By combining Eqs. (7) and (10) while eliminating the theoretical mean spacing length $\overline{SL}_{Theor}$, the theoretical mean clump length $\overline{CL}_{Theor}$ can be written in terms of the parameters known in the theoretical domain such as the original number of particles N, the number of clump/spacing pairs N_T as well as the mean diameter $\overline{D}_{Theor}$ in m as follows:

$$\overline{CL}_{Theor} = \left(\frac{N}{N_T}-1\right)\left(\frac{\overline{D}_{Theor}}{\ln\left(\frac{N}{N_T}\right)}\right) \tag{18}$$

Analogously, the spacing lengths are measured using

$$SL_i = v_i TS_i = b_S\frac{TS_i}{TF_i} \tag{19}$$

Note that the spacing velocity is assumed equal to the velocity of its preceding clump. The lumped parameter b_S was also characterised using measurement data. The mean value of the spacing lengths is

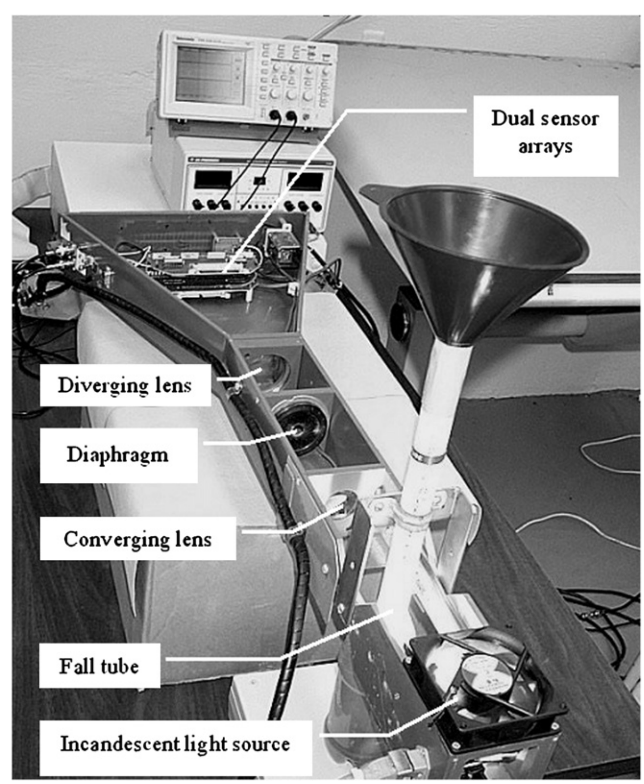

Fig. 5 – Photo of the time-of-flight device. The incandescent light source is covered with a fan for cooling, as it projects light through a slit in the fall tube onto dual sensor arrays after being magnified by the two lenses. Particles falling from the funnel through the tube form clumps whose shadow images are detected allowing the measurement of their velocity and lengths.

$$\overline{SL} = \frac{b_S}{N_T}\sum_{i=1}^{N_T}\frac{TS_i}{TF_i} \quad (20)$$

Solving for b_S gives

$$b_S = \overline{SL}\,\frac{N_T}{\sum_{i=1}^{N_T}\frac{TS_i}{TF_i}} \quad (21)$$

To calculate the value of b_S, the measured mean spacing length $\overline{SL}$ in m was replaced by its theoretical equivalent:

$$\hat{b}_S = \overline{SL}_{Theor}\frac{N_T}{\sum_{i=1}^{N_T}\frac{TS_i}{TF_i}} \quad (22)$$

By manipulating Eq. (7), the theoretical mean spacing length $\overline{SL}_{Theor}$ can be written in terms of parameters known in the theoretical domain such as the original number of particles N, the number of clump/spacing pairs N_T as well as the mean diameter $\overline{D}_{Theor}$ as follows:

$$\overline{SL}_{Theor} = \frac{\overline{D}_{Theor}}{\ln(\frac{N}{N_T})} \quad (23)$$

The sensor characteristic values b_C and b_S were determined using Eqs. (18) and (23) for all 30 datasets. The mean values obtained among these 30 datasets were assumed to be the most reliable and adopted as sensor characteristic constants. These constants ensure that the mean of the calculated mean diameters among all experiments was equal to the true mean diameter of the particles being 4.5 mm. The performance measure of calculating the mean diameter estimation method is now the variability of the calculated mean diameters among a range of flow densities, since the values of b_C and b_S are kept constant.

The mean particle diameter was calculated using the measured clump and spacing lengths from Eq. (15), respectively, Eq. (20), which include the characterisation parameters, b_C and b_S as follows:

$$\hat{\overline{D}} = \overline{SL}\ln\left(\frac{\overline{CL}}{\overline{SL}}+1\right) \quad (24)$$

3. Materials and methods

3.1. Experimental arrangement

Experiments were carried out using a funnel and fall-tube arrangement as shown in Fig. 3. The flow density in particles

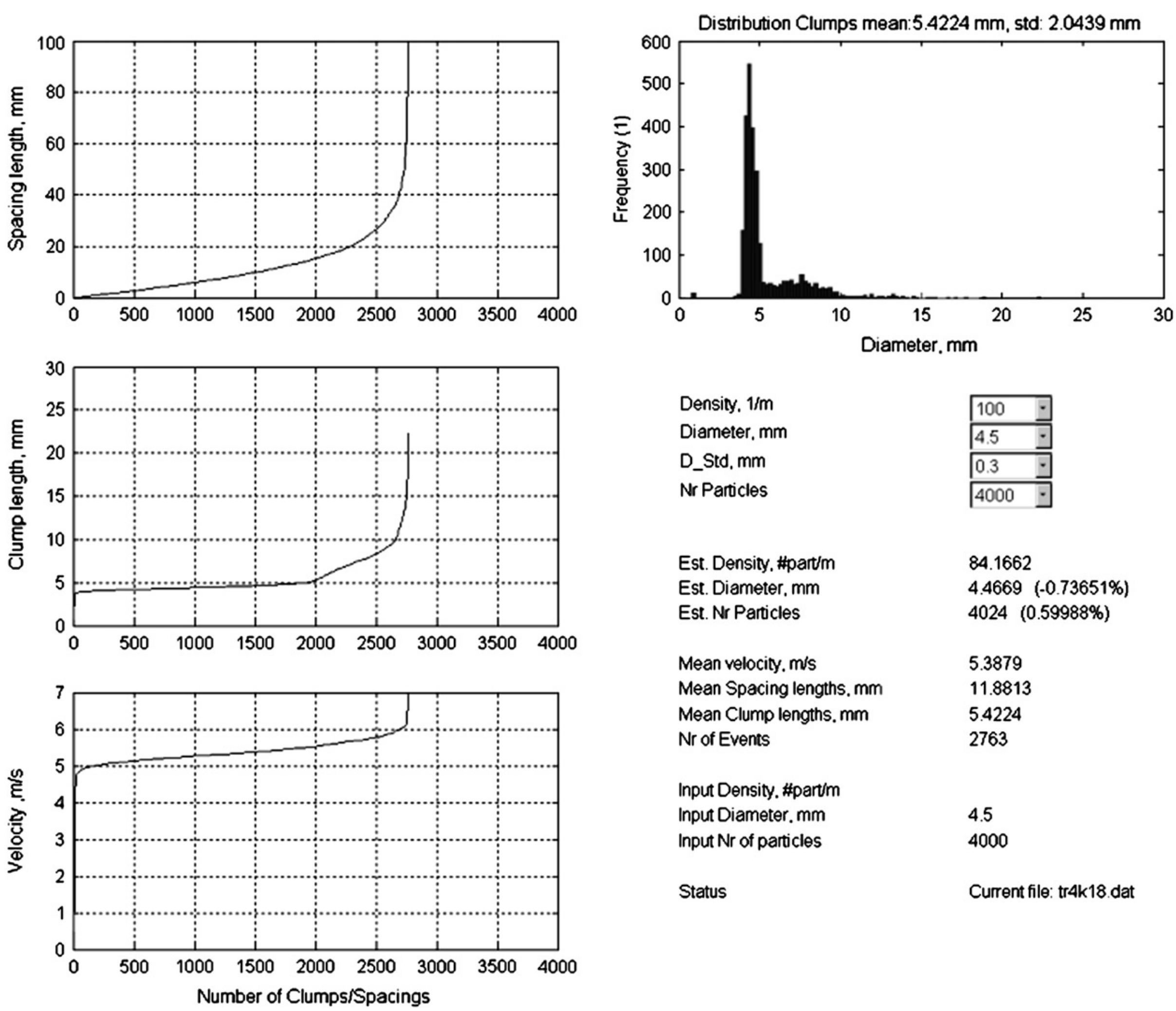

Fig. 6 – Screenshot of FlowSim 1.0, a MatLab® program capable of simulating Poisson processes, used for Poisson model validation based on measured clump and spacing lengths. The program also has a batch mode, which allows for taking multiple datasets into account. This feature was used for characterisation of the sensor.

per second is relatively independent of the drop height. In contrast, the density in particles per metre as used here is a function of drop height, since the particles accelerate in the tube. Since the mean diameter equation is based on the latter definition of flow density, it was useful to study the performance of the method using four drop heights. The binary signals (one of the two is illustrated) were recorded and the spacing and clump lengths measured using Eqs. (14) and (19) which include the two sensor characteristic constants b_C and b_S. For each experiment, 4000 spherical particles were dropped into the funnel in a swift motion after which the funnel emptied owing to gravity. The interruption times contained in the binary signals were measured using a counter/timer board with a clock rate of 20 MHz (PCI-6601, National Instruments, TX, USA).

3.2. *Time-of-flight device*

The optical time-of-flight device as shown in Fig. 5 consists from front to back of an incandescent halogen light source placed under a fan for cooling, a fall tube and funnel, a converging lens, a diaphragm, a diverging lens and dual sensor arrays in the background. Each sensor array contains 30 digital optical switches termed 'OptoSchmitts' (SDP8601, Honeywell, Scotland, UK) with a fall time of 15 ns and a rise time of 60 ns. All 30 switches in either array are connected in a logical AND function. In this way, when all OptoSchmitts receive light the array output is high, and if one or more are blocked the array output becomes low. This mechanism effectively produces two stacked light sensitive grids each consisting of 30 parallel optical interruption lines placed at a mutual lateral distance of approximately 0.63 mm. The lenses in the sensor magnify the image of the particles eight times, which enables the detection of small particles as small as 1 mm diameter with relatively large sensors (5 mm width). A complete description of the device can be found in Grift and Hofstee (1997).

3.3. *Analysis software*

The mean diameter calculations were carried out using a dedicated program named FlowSim, written in MatLab® (2007). This program contained a module capable of simulating Poisson driven particle flow, a matching module for model validation as well as batch processing of data files, which enable characterisation. Fig. 6 shows a screenshot from the program.

4. Results

To assess how closely the flow resembles a theoretical Poisson process, the number of clump/spacing pairs was computed as a function of the flow density. The relationship between the theoretical number of clump/spacing pairs and the flow density was obtained by substituting Eq. (4) into Eq. (2) leading to

$$N_T = Ne^{-\lambda_{\mathrm{Theor}} \times \overline{D}_{\mathrm{Theor}}} \quad (25)$$

Here N was set to 4000 initial particles, $\overline{D}_{\mathrm{Theor}}$ to 4.5 mm, and the flow density ranged from 60 particles m^{-1} to 180 particles m^{-1}. When the counted number of clump/spacing pairs is compared to their theoretical counterparts, shown as a solid line in Fig. 7, it is clear that they are closely correlated. However, at the highest flow densities (measured at a small distance from the funnel) the number of clump/spacing pairs N_T is consistently lower than the theoretical numbers. It was concluded that here the Poisson assumption is not viable, since the flow had not made the complete transition from

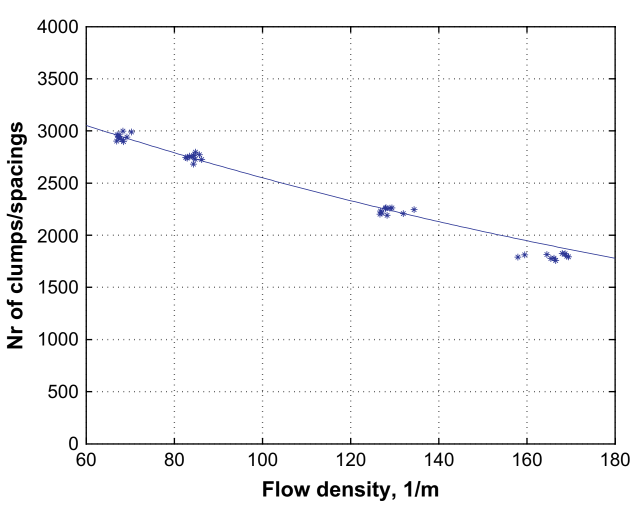

Fig. 7 – Counted number of clump/spacing pairs from experiments with 4000 identical 4.5 mm particles. The solid line represents the theoretical number of clump/spacing pairs as a function of flow density. The highest density dataset around 160 particles m^{-1} was considered to be non-Poisson and excluded from the characterisation and the mean diameter estimation process.

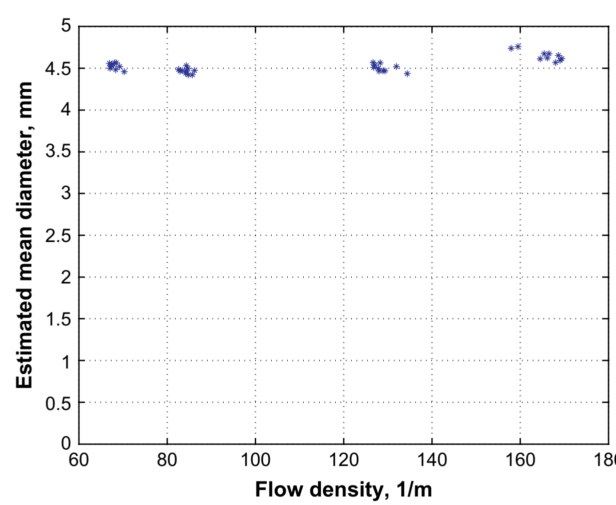

Fig. 8 – Estimated diameters of particles from experiments with 4000 identical 4.5 mm particles. The mean of the estimates was 4.50 mm (after characterisation) with a standard deviation of 0.044 mm (coefficient of variation 1%) based on the 30 datasets with flow densities from approximately 67 particles m^{-1} to 167 particles m^{-1}. The highest density datasets at approximately 160 particles m^{-1} were assumed non-Poisson driven.

a relatively fixed packing in the funnel to a random arrival process. Therefore, the 10 datasets with the highest flow densities were not used in the characterisation process. The values of b_C and b_S based on the remaining 30 datasets were found to be 0.79 mm and 1.21 mm, respectively.

After characterisation, the mean diameters of the particles in the flow were calculated using Eq. (24). Fig. 8 shows the resulting calculated diameters as a function of flow density, which are close to the true 4.5 mm for the lower flow densities. The diameter of the 4.5 mm particles was estimated from the 30 datasets assumed to form a Poisson process, with a mean value of 4.50 mm and a standard deviation of 0.044 mm (1% coefficient of variation). The maximum and minimum values were 4.57 mm (+1.6% error) and 4.42 mm (−1.8% error), respectively. The high density datasets around 160 particles m^{-1} are merely shown in the figure to illustrate the effect of a non-Poisson process on the estimated diameters.

5. Conclusions

A Poisson model was used to derive a theoretical method for estimating the mean diameter of particles free-falling in a tube. After characterisation of a time-of-flight sensor and estimation of the mean particle diameters, the following conclusions were drawn:

1. The flow of free-falling particles in a tube closely resembles a Poisson process, unless the fall distance from a funnel is too short and there is insufficient time for the flow to be transformed into a fully random arrival process.
2. The optical time-of-flight sensor configuration is adequate to estimate mean particle diameters in granular flow after characterisation.
3. The mean diameter of the particles can be estimated accurately after characterisation: for 4.5 mm diameter particles it is consistent among flow densities varying from approximately 67 particles m^{-1} to 167 particles m^{-1}.

The limitations of the method are that it has only been tested for identical particles in a relatively low density flow regime. Although theoretically the method should work for any particle distribution, this has not been tested in practice. Secondly, the sensor arrangement contains three independent types of error, and to make this method truly universal an improved sensor with lower errors is required. Thirdly, the method fails when the flow is so dense that no spacings among the clumps can be detected. In this case it is advised to accelerate the flow such that a lower flow density (in particles per metre) is obtained, or the flow can be split into subflows of lower density.

Intuition indicates that every granular flow will over time become a random arrival process (Poisson process) due to random disturbances experienced by the flow elements during their travel. However, in many applications, the flow elements are initially ordered such as cars waiting in line for a traffic light, grains and fruits transported on compartmentalised conveyor belts, and as in this paper, particles in a fixed location in a funnel. In these cases the flow needs time to make the transition to a Poisson flow and therefore it is advised to place the sensor ‘far enough’ from the initial deterministic state. How fast granular materials transform into a Poisson flow, under what conditions, and why, is currently unknown.

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