SPREAD PATTERN ANALYSIS TOOL (SPAT):
I. DEVELOPMENT AND THEORETICAL EXAMPLES

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ABSTRACT. *A custom Windows-based program called SPAT was written in MatLab (1999) to theoretically investigate the overall quality of spread patterns. It computes overlapped patterns, spread pattern characteristics, and produces cv-swath width and rate-swath width graphs. Pattern shapes that are often seen in practice such as Triangles, Trapezoids, M- and Crown shapes were analyzed. The effect of skewness, a pattern distorting factor in aerial as well as ground-based application, was studied as well. Traditionally, the quality of a spread pattern is defined in terms of the measured application rate (% deviation from the target rate) and the uniformity (expressed as the statistical coefficient of variation, cv). Both factors are not constants, but are functions of the swath width which makes their practical value disputable. In this study a new descriptor called "robustness" was defined. Robustness is a value based on the average cv-values over a range of potential swath-widths. A robust pattern allows the applicator to change the swath width (and the application rate), and be confident that no damage to the crop will occur. The most robust pattern found was a Gaussian shape, also known as a 'bell curve'. The second most promising pattern found was a symmetric triangle with the target rate value in the center. Trapezoids were found to be acceptable and even more efficient than triangles, although too much emphasis on large swath widths can make the pattern non-robust. Skewness was found to be a major factor in producing non-robust patterns; it tends to increase the cv over the whole range of useful swath widths. Although SPAT was written for analysis of aerial spread patterns, the overall results of this study can be applied to ground based application.*
Keywords. Fertilizer spread pattern, Spread pattern robustness, MatLab.

The objective of aerial as well as ground-based fertilizer application is to spread material at a preset rate and an acceptable uniformity, while minimizing the total travel distance and operation time. Testing of fertilizer application equipment is performed using a collection tray method, according to ASAE Standard S386.2 (1999) for aerial application and S341.3 (1999) for ground-based application. During the test, fertilizer is dispersed typically over 25 trays aligned perpendicularly to the path of travel. After spreading, each tray contains a sample representing the amount of material per tray area (gm^{-2}). These data give an indication of the pattern shape. In actual field operations, several consecutive passes are made with overlaps in either Back&Forth mode (adjacent passes in the opposite direction of travel) or RaceTrack mode (adjacent passes in same direction of travel). For a pattern test to predict field results, this overlapped pattern must be computed by a computer program along with two important characteristics, the rate ($kg\ ha^{-1}$) and the cv (%), both as functions of the swath width. The rate is essentially the mean of the overlapped pattern while the uniformity is quantified using the statistical coefficient of variation (cv), defined as the standard deviation divided by the mean. Whether an overlapped pattern is "acceptable" depends on

the value of cv at a certain swath width. A rule of thumb often used in aerial application is that a cv value lower than 15% is assumed not to cause damage to the crop (Gardisser, 1993). This value is used throughout all analyses in this article.

To minimize the total travel distance, the operator will try to use the maximum swath width that results in a pattern of acceptable uniformity. In aerial application the rate is controlled by a gate, which ideally feeds material into the spreader ducts such that the desired pattern emerges on the ground. If during tests the pattern deviates from the desired shape, the functioning of the gate is checked and the distances between the vanes that separate the ducts are adjusted to correct the problem.

A scenario that often occurs during testing is an aircraft producing a pattern of acceptable uniformity at a certain swath width, but the desired rate is not obtained. Sometimes the desired rate is achieved but the uniformity is unacceptable. The main reason for this trade-off is the fact that pilots have traditionally had little control over the application rate since there is no direct feedback of the material flow in the spreader. Grift and Hofstee (1997) have developed a sensor that can be used to measure the fertilizer flow through the ducts, which can alleviate this problem.

The most efficient spreading approach would be to have the spreader always work at the maximum material flow (without overloading it) and adjust the swath width to set the application rate. This approach is only valid if the uniformity remains acceptable, even when the swath width is changed. A pattern that has acceptable uniformity for a large range of swath widths is called "robust". A robust pattern is inherently forgiving, changes in swath width,

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either deliberate or due to flight path errors will not have great effects on the uniformity.

The objectives of this study were to:

- Investigate various hypothetical shapes to obtain the most robust spread pattern that is realizable in practice.
- Investigate the role of skewness in producing non-robust patterns.

MATERIALS AND METHODS

All shapes studied were defined as a 25-point sequence and stored in an Excel spread sheet. Each row contained administrative fields such as 'Date', 'Run Number', 'Pilot name', 'Aircraft ID', 'Spreader Type', 'Target Rate', and 25 fields containing the data points. The same data format can be used for actual field testing, by replacing the 25 data points by the actual weight of the collected samples and entering realistic values in the administrative fields. During ground based testing, trays are usually left out where the machinery wheels pass. In this case the missing values can be interpolated in the spread sheet and SPAT can be used to analyze the data.

The basic shapes that were investigated are:

1. Gaussian (see fig. 1). The Gaussian shape was chosen to study a smooth form without discontinuities.
2. Wide Triangle (see fig. 3). Triangles are attractive shapes because, in theory, when overlapped at half the spread width, they form a perfectly uniform overlapped pattern.
3. Narrow Triangle (see fig. 4). A narrow triangle was studied to compare with the Wide Triangle.
4. Trapezoids. These shapes are very popular since a wide swath width can be realized and theoretically, they can produce a cv of zero when properly overlapped. Four trapezoids were studied to determine the limit where trapezoids become non-robust. They are characterized by the ratio of the horizontal leg divided by the overall spread width, being 3/12 (see fig. 5), 4/12, 5/12, and 6/12 (see fig. 6).
5. Skewed Trapezoid (see fig. 7). Skewness is often caused by side wind and in aerial application by gates that do not open evenly. In ground-based application the air distortion caused by the impeller can contribute as well. Skewness can have very negative effects on the uniformity especially in Back&Forth operation.
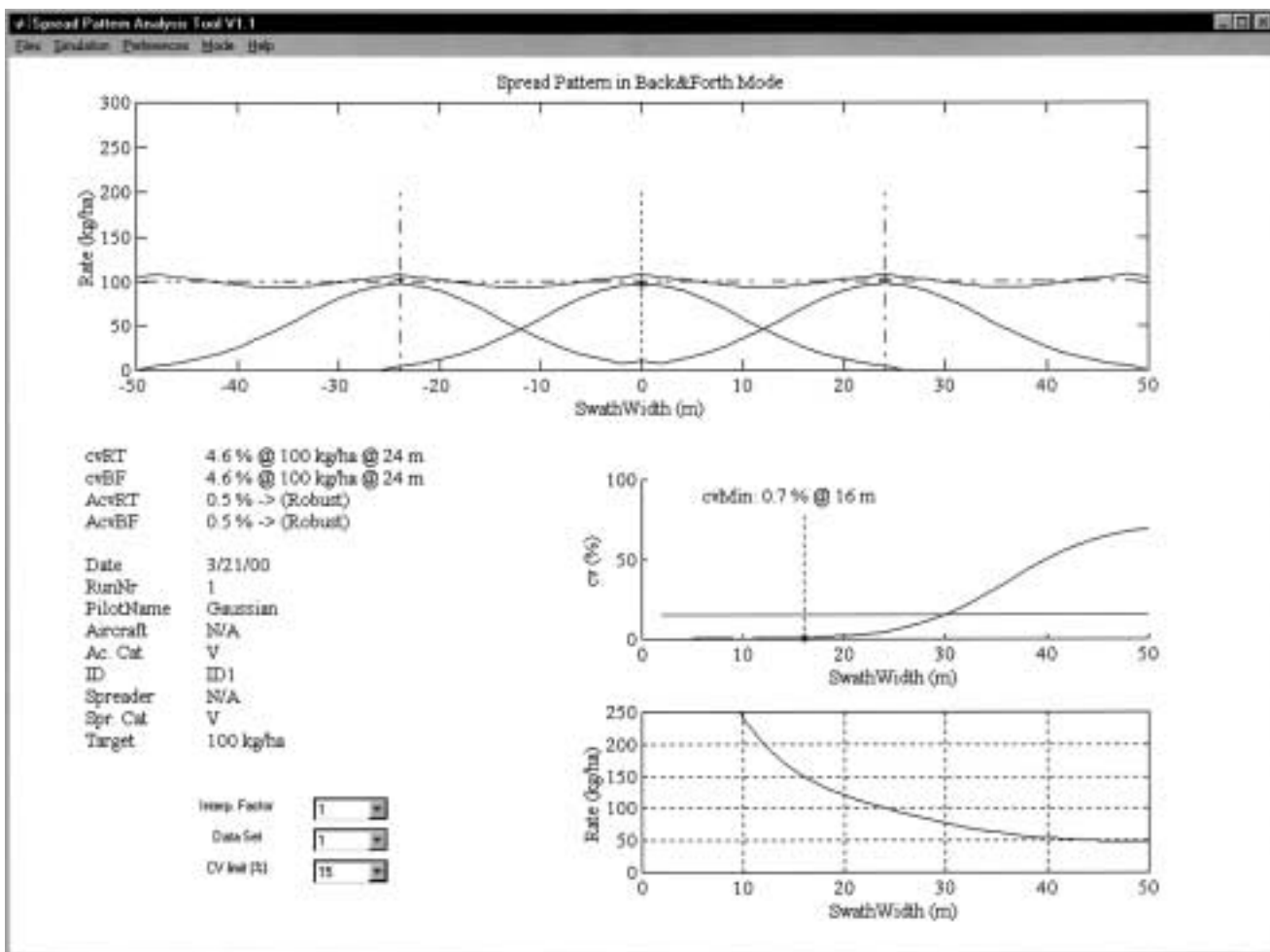

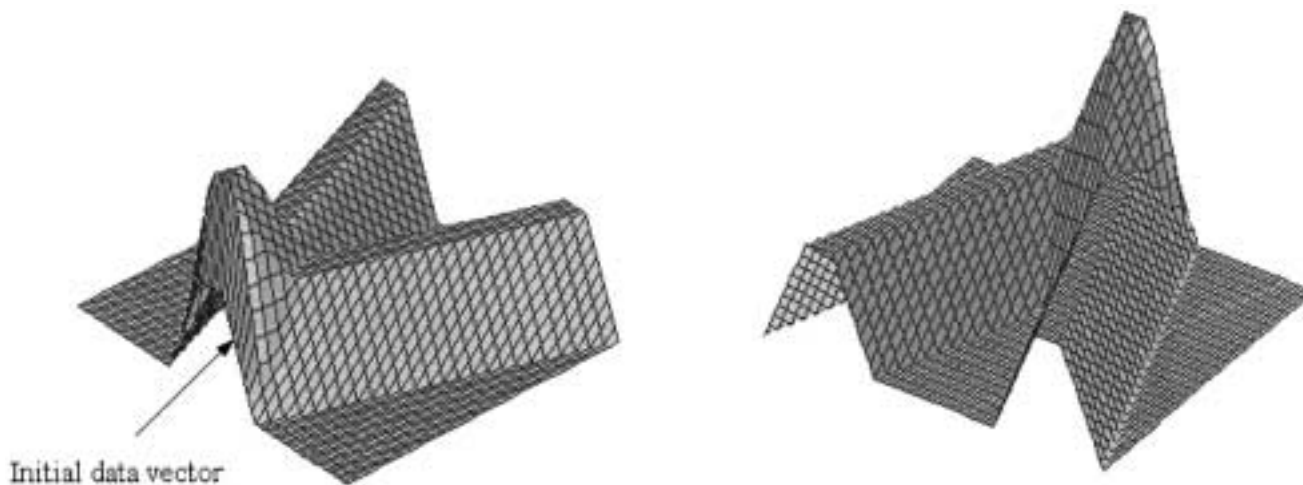Figure 1–Spread Pattern Analysis Tool (SPAT).

Initial data vector

Figure 2–Principal overlap matrix for trapezoidal pattern.

6. Crown (see fig. 8). A Crown shape occurs when some spreader ducts carry more material than others. It has a negative effect on the spread pattern and is often combined with skewness.

7. M-shape (see fig. 9). In aerial application, covering larger swaths requires more powerful engines and larger spreaders, since the objective is to spread the material as wide as possible. Bansal (1997), has shown that adding ducts to a spreader does not necessarily result in a wider spread width. Too much emphasis on a wide swath often leads to an M-shape. Also, ground-based pendulum spreaders sometimes suffer from M-shaped patterns (Parish, 1999).

8. Skewed M-shape (see fig. 10). This pattern represents the causes of the M-shape in combination with the main cause of non-uniformity, skewness.

In this study the patterns were scaled, such that the target rate (assumed 100 kg ha^{-1}) was always present at a swath width of 24 m to simplify comparing pattern shapes. This does not affect the cv-swath width curve, since the coefficient of variation is by definition invariant to scaling.

PROGRAM SPREAD PATTERN ANALYSIS TOOL (SPAT)

Earlier spread pattern analysis tools were written in the DOS environment (Parish, 1987). In contrast, SPAT is a Windows-based application, written in MatLab (1999), a tool for scientific computation. The advantage of MatLab is the fact that the elementary computational unit is a matrix rather than a scalar. This feature makes it very well suited for computation of overlapped spread patterns, since this is essentially the addition of space-shifted data vectors. The spread pattern shapes were originally defined in Excel and SPAT retrieved the data points and administrative information from Excel through a Dynamic Data Exchange (DDE) link. Spread pattern characteristics computed in SPAT were returned to the Excel spread sheet using the same DDE link. The Excel spread sheet can contain multiple data sets. SPAT automatically determines the number and the user can select them individually. SPAT
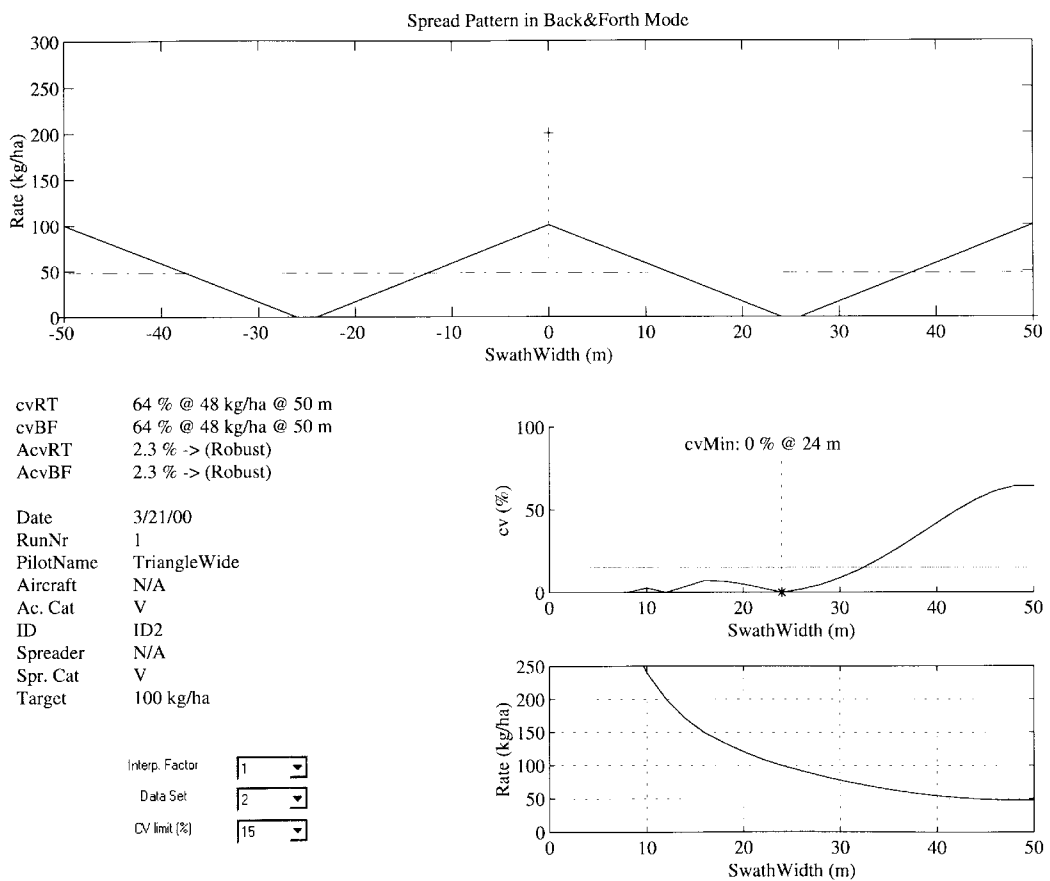
uses a Graphical User Interface, shown in figure 1 (as an example, a Gaussian shape is shown, overlapped at a swath width of 24 m).

The top graph in figure 1 shows the measured pattern shape (Gaussian) in the center and the overlapped spread pattern [application rate (kg ha^{-1}) versus swath width (m)]. In addition to the measured pattern, two adjacent patterns shifted by the width of the swath are shown. The vertical dash-dotted lines represent the actual swath widths and the horizontal dash-dotted line represents the average application rate [in this case 100 (kg ha^{-1})]. The tray spacing in SPAT was set to 2 m, which is why a 25-point spread pattern has a width of 50 m. In future versions this value will be variable.
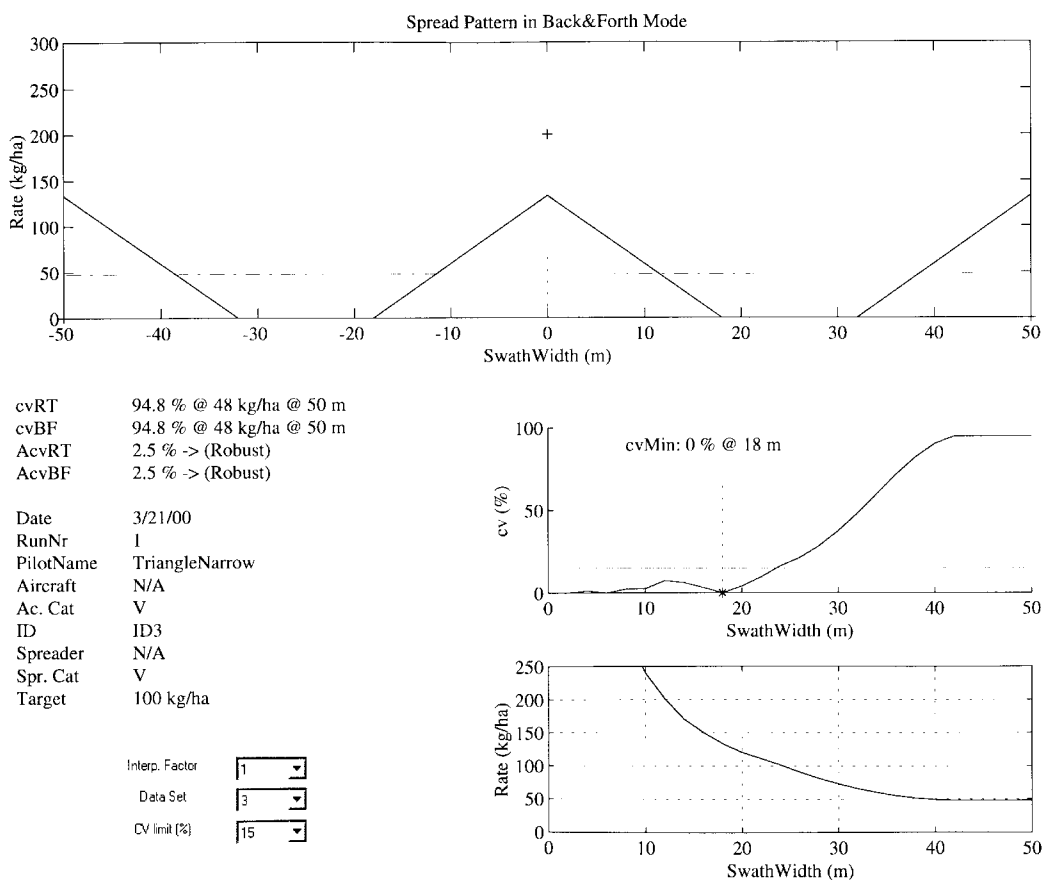
The data block at the bottom left of figure 1 (from 'Date' through 'Target') is administrative data retrieved from the Excel spread sheet. The text block above it represents the actual values of the coefficient of variation for RaceTrack (cvRT) and Back&Forth modes (cvBF). These two values correspond to the actual overlapped pattern as shown in the top graph of figure 1.

The two values AcvRT (Average cv in RaceTrack mode) and AcvBF (Average cv in Back&Forth mode) represent the robustness of the pattern. They are computed as the mean cv in a swath width interval of zero to the maximum swath width where a minimum occurs. The robustness was defined as such under these arguments:

- The swath width range over which the robustness is computed must be computed in a consistent manner, rather than having the user select it. For **all** spread patterns the cv monotonically increases for large swath widths and tends to a constant where no overlapping occurs (see fig. 1, center right graph and the similar graphs in figs. 3 through 10). Since it does not make sense to use this range, robustness was based on cv's in the complementary range, from zero to the minimum encountered at the maximum swath width.

- The spread patterns of a large aircraft with a wide spread width should have the same robustness as that of a smaller aircraft, as long as the basic shape of the spread pattern is the same. This is why the robustness

Spread Pattern in Back&Forth Mode

Figure 3–The SPAT output for a Wide Triangle pattern.

cvRT	64 % @ 48 kg/ha @ 50 m
cvBF	64 % @ 48 kg/ha @ 50 m
AcvRT	2.3 % -> (Robust)
AcvBF	2.3 % -> (Robust)
Date	3/21/00
RunNr	1
PilotName	TriangleWide
Aircraft	N/A
Ac. Cat	V
ID	ID2
Spreader	N/A
Spr. Cat	V
Target	100 kg/ha

Interp. Factor 1
Data Set 2
CV limit (%) 15

cvMin: 0 % @ 24 m

Spread Pattern in Back&Forth Mode

Figure 4–The SPAT output for a Narrow Triangle pattern.

cvRT	94.8 % @ 48 kg/ha @ 50 m
cvBF	94.8 % @ 48 kg/ha @ 50 m
AcvRT	2.5 % -> (Robust)
AcvBF	2.5 % -> (Robust)
Date	3/21/00
RunNr	1
PilotName	TriangleNarrow
Aircraft	N/A
Ac. Cat	V
ID	ID3
Spreader	N/A
Spr. Cat	V
Target	100 kg/ha

Interp. Factor 1
Data Set 3
CV limit (%) 15

cvMin: 0 % @ 18 m

Spread Pattern in Back&Forth Mode

cvRT   59.3 % @ 48 kg/ha @ 50 m
cvBF   59.3 % @ 48 kg/ha @ 50 m
AcvRT   3 % -> (Robust)
AcvBF   3 % -> (Robust)

Date   3/21/00
RunNr   1
PilotName   Trapezoid (3/12)
Aircraft   N/A
Ac. Cat   V
ID   ID4
Spreader   N/A
Spr. Cat   V
Target   100 kg/ha

Interp. Factor   1
Data Set   4
CV limit (%)   15

cvMin: 0 % @ 30 m

Figure 5–The SPAT output for a Trapezoid (3/12) pattern.

Spread Pattern in Back&Forth Mode

cvRT   49.9 % @ 48 kg/ha @ 50 m
cvBF   49.9 % @ 48 kg/ha @ 50 m
AcvRT   7.7 % -> *Non-Robust*
AcvBF   7.7 % -> *Non-Robust*

Date   3/21/00
RunNr   1
PilotName   Trapezoid (6/12)
Aircraft   N/A
Ac. Cat   V
ID   ID5
Spreader   N/A
Spr. Cat   V
Target   100 kg/ha

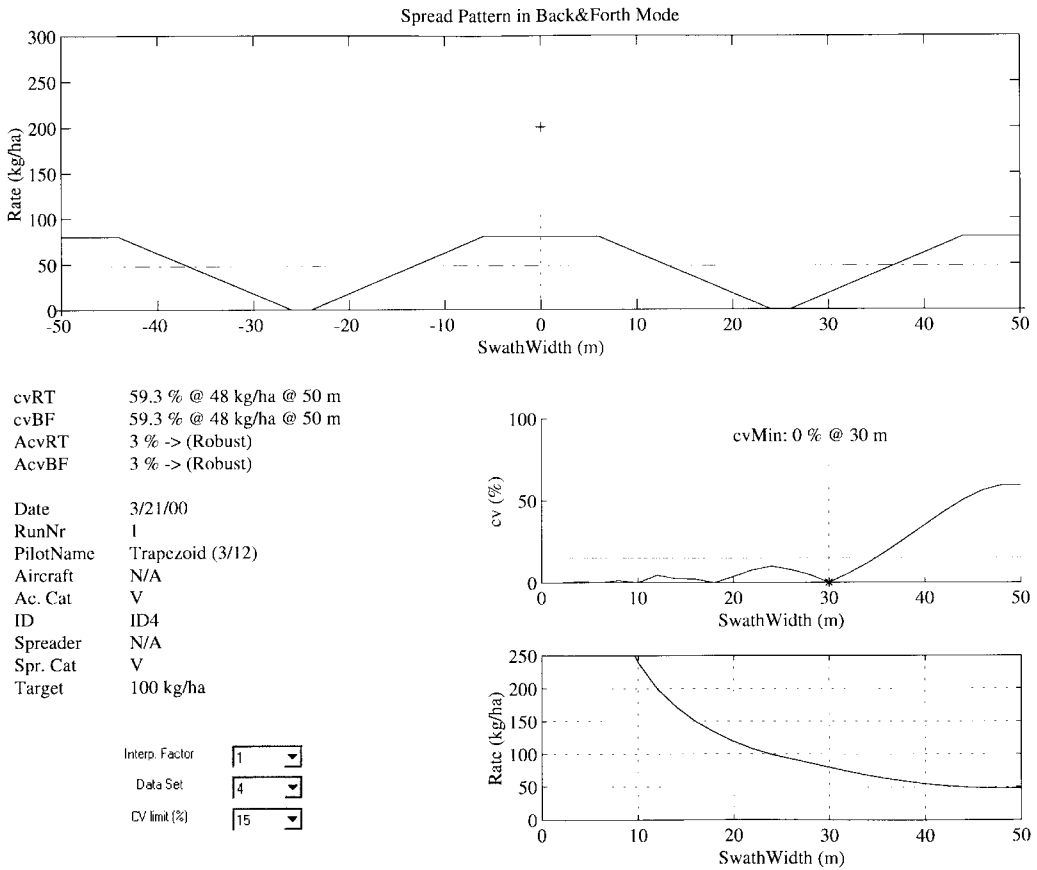Interp. Factor   1
Data Set   7
CV limit (%)   15

cvMin: 0 % @ 36 m
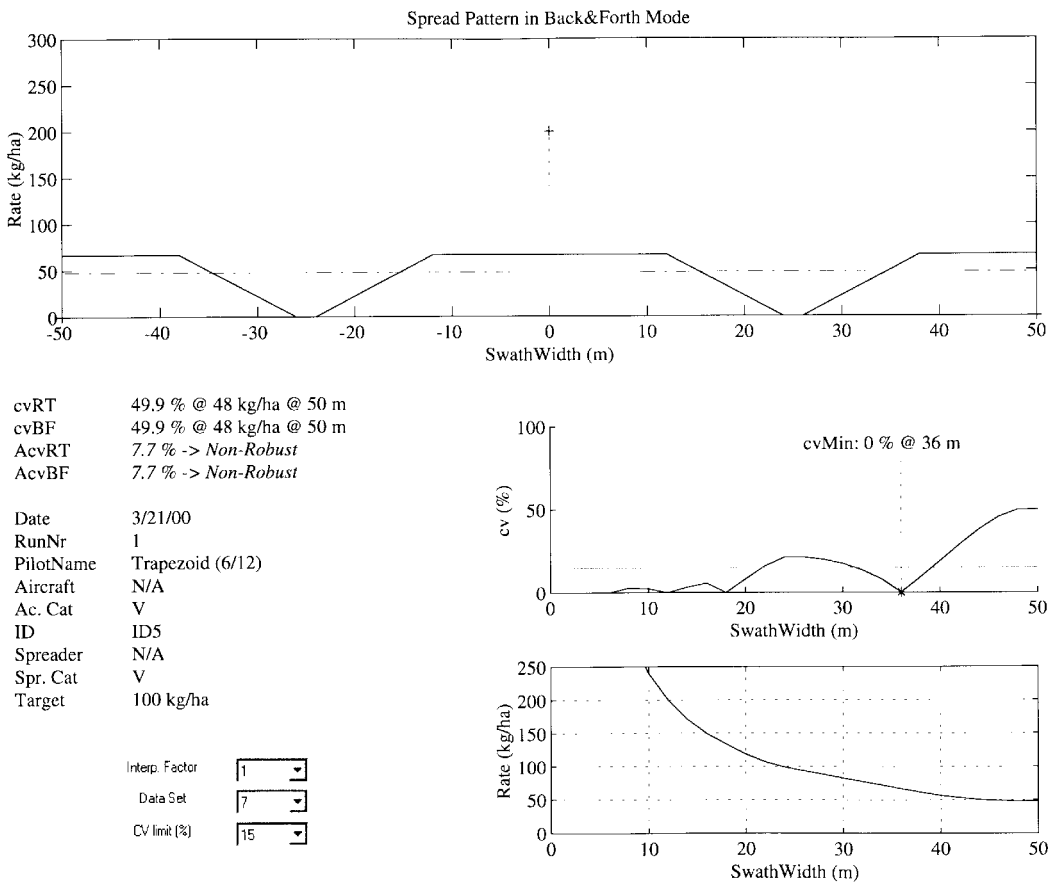
Figure 6–The SPAT output for a Trapezoid (6/12) pattern.

Spread Pattern in Back&Forth Mode

cvRT   57.5 % @ 48 kg/ha @ 50 m
cvBF   57.5 % @ 48 kg/ha @ 50 m
AcvRT   *11.9 % -> Non-Robust*
AcvBF   *13.7 % -> Non-Robust*

Date   3/21/00
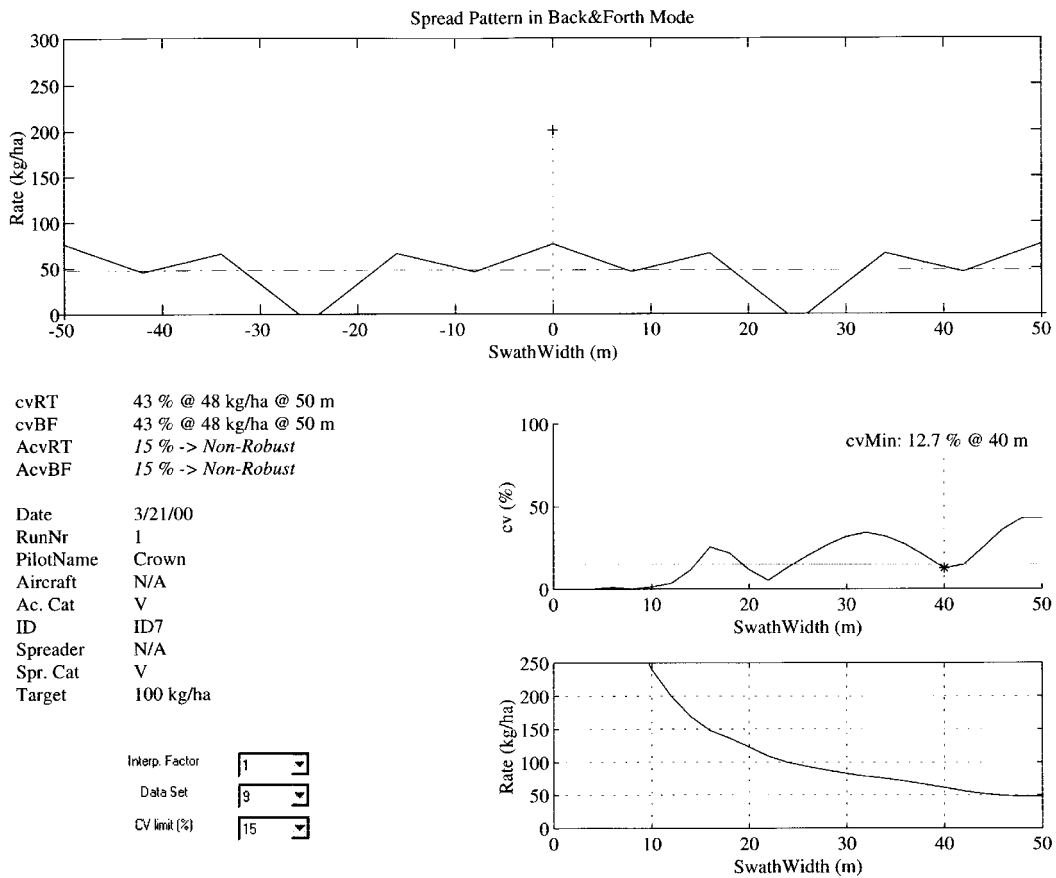RunNr   1
PilotName   TrapezoidSkewed
Aircraft   N/A
Ac. Cat   V
ID   ID6
Spreader   N/A
Spr. Cat   V
Target   100 kg/ha

Interp. Factor   1
Data Set   8
CV limit (%)   15

cvMin: 13.4 % @ 36 m
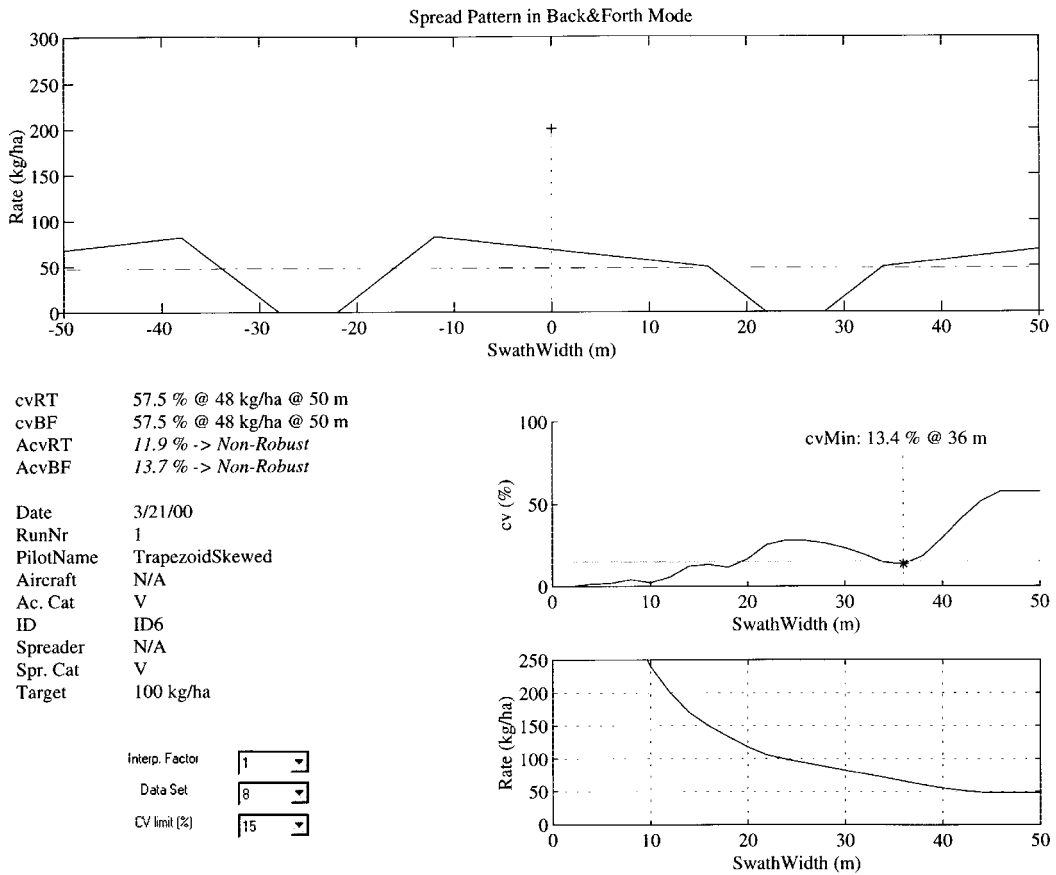
Figure 7–The SPAT output for a Skewed Trapezoid pattern.

Spread Pattern in Back&Forth Mode

cvRT   43 % @ 48 kg/ha @ 50 m
cvBF   43 % @ 48 kg/ha @ 50 m
AcvRT   *15 % -> Non-Robust*
AcvBF   *15 % -> Non-Robust*

Date   3/21/00
RunNr   1
PilotName   Crown
Aircraft   N/A
Ac. Cat   V
ID   ID7
Spreader   N/A
Spr. Cat   V
Target   100 kg/ha

Interp. Factor   1
Data Set   9
CV limit (%)   15

cvMin: 12.7 % @ 40 m

Figure 8–The SPAT output for a crown pattern.

Spread Pattern in Back&Forth Mode

cvRT   42.1 % @ 48 kg/ha @ 50 m
cvBF   42.1 % @ 48 kg/ha @ 50 m
AcvRT   *17.8 % -> Non-Robust*
AcvBF   *17.8 % -> Non-Robust*

Date   3/21/00
RunNr   1
PilotName   M-Shape
Aircraft   N/A
Ac. Cat   V
ID   ID8
Spreader   N/A
Spr. Cat   V
Target   100 kg/ha

Interp. Factor   [1 ▼]
Data Set   [10 ▼]
CV limit (%)   [15 ▼]

cvMin: 13.6 % @ 42 m
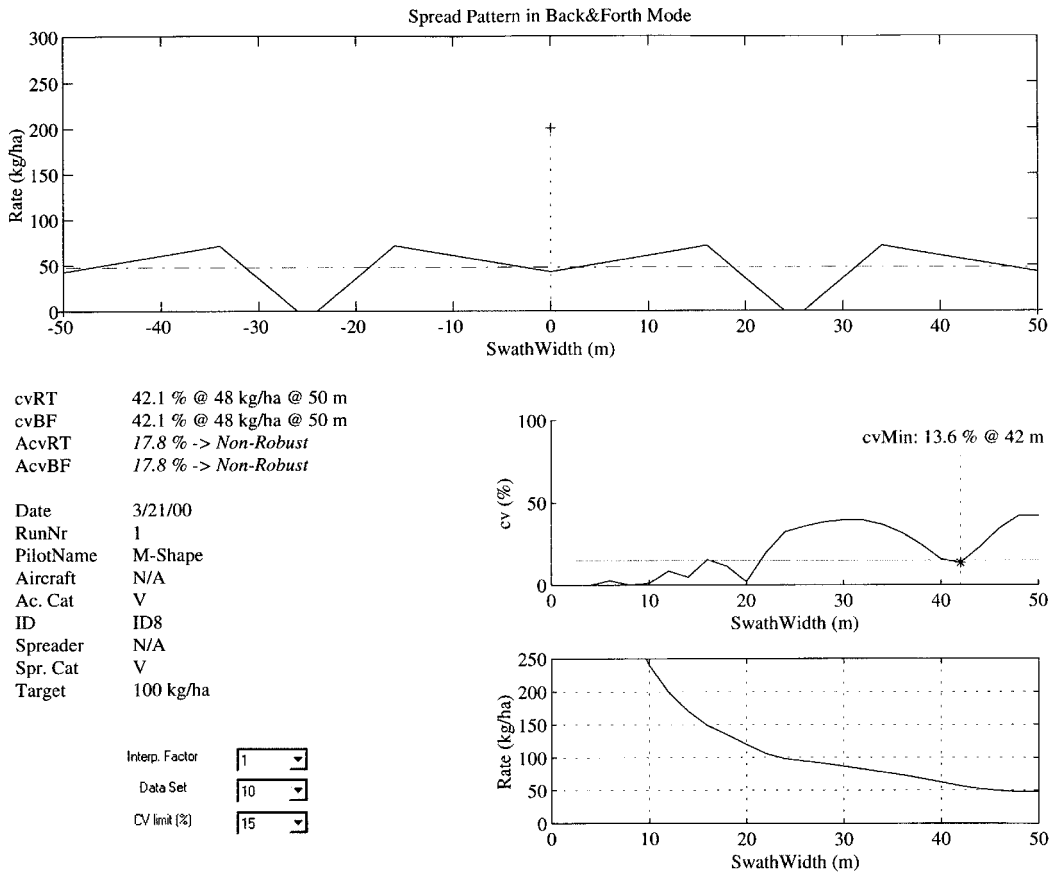
Figure 9–The SPAT output for a M-shape pattern.

Spread Pattern in Back&Forth Mode

cvRT   69.9 % @ 48 kg/ha @ 50 m
cvBF   69.9 % @ 48 kg/ha @ 50 m
AcvRT   *18.3 % -> Non-Robust*
AcvBF   *21.2 % -> Non-Robust*

Date   3/21/00
RunNr   1
PilotName   M-ShapeSkewed
Aircraft   N/A
Ac. Cat   V
ID   ID9
Spreader   N/A
Spr. Cat   V
Target   100 kg/ha

Interp. Factor   [1 ▼]
Data Set   [11 ▼]
CV limit (%)   [15 ▼]
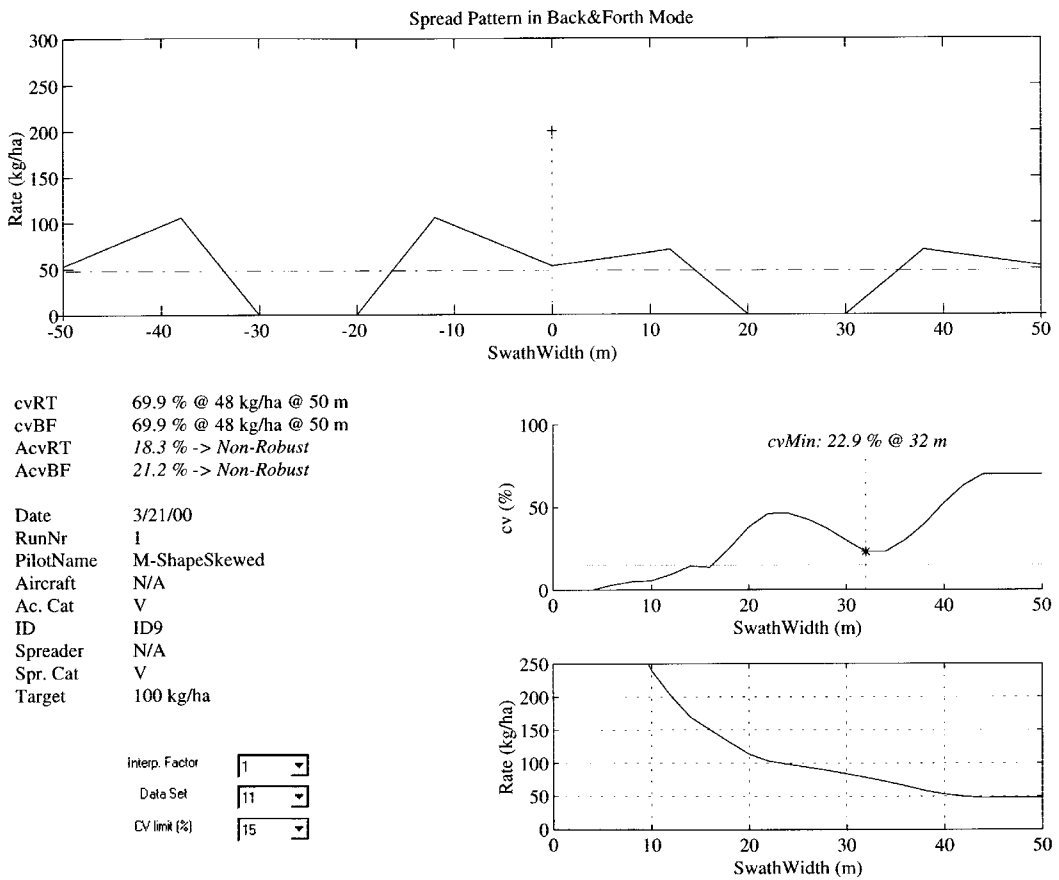
cvMin: 22.9 % @ 32 m

Figure 10–The SPAT output for a Skewed M-shape pattern.

was defined as a relative measure: the mean of the cvs in the swath width range described earlier.

The cv-swath width curve itself is plotted in the center right position. The horizontal line in this plot represents a cv that is assumed not to cause damage to the crop. The default value is 15% but it can be changed to a value that represents other crops, climatic conditions and materials.

The bottom right plot in figure 1, shows the application rate as a function of swath width. In this example, the selected swath width was 24 m, leading to a cv of 4.6% for both Back&Forth and RaceTrack modes (the cvs of symmetrical shapes are the same in both modes). The cv-swath width plot shows that the cv remains lower than 15% for a large range of swath widths. The pilot could use a 30 m swath width [which lowers the application rate to 80 (kg ha^{-1})] and still maintain a cv lower than 15%. Also, all lower swath widths are producing patterns with less than 15% cv. In conclusion, the Gaussian pattern is extremely robust.

The pop-up menu labeled 'Interp.Factor' allows the user to interpolate the data (up to a factor 5) to obtain a smoother plot. The pop-up menu labeled 'Data Set' is used to select data sets if the Excel spread sheet contains multiple sets. The pop-up menu 'CV limit (%)' allows the user to set the value of cv where no damage to the crop may be expected.

PROGRAM FUNCTIONALITY

The pull-down menus available in the program, have the following functions:

Files

Link File	Opens a file selector dialog box from which the Excel spread sheet can be selected.
Exit	Exits the program.

Simulation

Run	Computes the overlapped pattern for a selected data set.
Stop	Interrupts a running overlap computation.
Select	The user can select a swath width corresponding to a certain rate/uniformity combination with the mouse. The program then renders the associated spread pattern with numerical pattern characteristics.

Preferences

Continuous	Runs all available data sets from the Excel spread sheet.
Add Plots	When the check box is active, the program overlays all consecutive plots. This feature allows for comparison of multiple spread runs. In practice, it is useful to observe the effects of adjustments to the spreader.
Graph	Switches graphical output on/off.
Centered	Shifts a pattern such that its centroid is located at the center tray location. This is used in Back&Forth flight mode to compensate for off-center aircraft

passage during tests. In ground based application it is usually turned off. In the two skewed patterns in this study it is turned off as well.

Animation	When selected, the overlapped pattern is shown for every increment (one tray spacing for non-interpolated data).

Mode

RaceTrack	Computes overlapped patterns in RaceTrack mode.
BackForth	Computes overlapped patterns in Back&Forth mode.

Help

About SPAT	Produces an information screen.

COMPUTATION OF OVERLAPPED PATTERNS

The input to the overlapping algorithm was a single 1*25 (number of trays) data vector containing "tray content weight" values from the Excel spread sheet. To calculate the overlapped pattern, first a matrix was computed that contained all possible overlaps (principal overlaps) for RaceTrack flight paths in increments of one tray spacing. The initial data vector (the spread pattern) was stored in the first row of the overlap matrix (pattern 1). The first principal overlap occurs at a swath width of one tray spacing from the center (pattern 2). The final principal overlap is the case where the adjacent patterns are no longer overlapping (pattern 25). In figure 2, on the left, the principal overlap matrix is shown in three dimensions (as an example a trapezoidal shape was chosen). The basic pattern itself (no overlap) is the first pattern shown on the left. All other possible overlaps are shown progressively to the right. The last pattern (extreme right) is the situation where no more overlap is present (the original trapezoid would fit exactly between the two outer trapezoids without overlap). The right hand figure is the same matrix, shown from the opposite direction.

An overlapped spread pattern at an arbitrary swath width was computed by adding the initial data vector (the spread pattern) to the principal overlaps for a given swath width. For example, for a swath width of five tray spacings, the initial data vector was added to principal overlaps 6, 11, 16, and 21.

The Back&Forth spread pattern computations are basically the same as their RaceTrack counterparts, except that for every other overlap, the vector needs to be mirrored and then added. Again, for an overlap of five tray spacings, the initial data vector was added to principal overlaps 6*, 11, 16*, and 21, where the asterisk indicates that mirrored vectors were used.

The cv (%) and the rate (kg/ha) were computed as a function of swath width by overlapping the pattern for all swath widths in increments of one tray spacing.

Once these are shown on the screen, the user can select a swath width of interest with the mouse (for instance a swath width that yields the desired application rate, combined with an acceptable cv) and the program shows the corresponding spread pattern along with numerical pattern characteristics. The latter are also transferred back into the Excel spread sheet through the DDE link.

The program can interpolate the data by a factor of 3 or 5. In the case of 3, the program converts the initial 1*25 data vector to an interpolated 1*75 data vector. Although the dimensions of the overlap matrices change, the computation procedure remains fundamentally the same.

For users who prefer English units, the program can be easily adapted, only two conversion factors are involved, gram per square meter (kg/ha) to pounds per acre (lb ac^{-1}) and meters (m) to feet (ft). In future versions, the program will have a simple button that allows the user to switch between metric and English units.

ANALYSIS OF HYPOTHETICAL SPREAD PATTERNS

The best shape found was the Gaussian shape (bell curve) and will be discussed first. The other shapes were divided into desirable shapes (triangles and trapezoids) and undesirable shapes [skewed trapezoid, M-shape (with and without skewness) and a Crown shape].

GAUSSIAN

A Gaussian spread pattern was discussed earlier (see fig. 1). From the cv curve, it is clear that this is an excellent shape since the average cv remains very low for a large range of swath widths. Although the smoothness of the Gaussian suggests that the cv-swath width curve must be continuous, there is a minimum at 16 m, caused by the inevitable truncation at the ends of the Gaussian itself.

DESIRABLE SHAPES: TRIANGLES AND TRAPEZOIDS

The desirable shapes are shown in figures 3 through 6.

Wide Triangle (fig. 3). The Wide Triangle shape is attractive because it has a distinct minimum where the cv reaches zero. This point occurs when the swath width is equal to half the width of the original spread pattern, which theoretically results in a perfectly uniform spread pattern (cv = 0% @ 24 m swath width). When compared with the Gaussian, this pattern shows a "hump" in the cv curve which stays well below the 15% line. This is an excellent robust pattern.

Narrow Triangle (fig. 4). The cv-rate curve of the Narrow Triangle pattern showed a high similarity with the Wide Triangle pattern, as it too has a distinct minimum where the cv reaches zero (cv = 0% @ 18 m swath width). The only difference is a shift to the left in the cv-swath width curve, which implies that this pattern has the same potential for producing excellent results, be it at lower swath width values.

Trapezoid Narrow (3/12) (fig. 5). The Narrow Trapezoidal pattern also has a distinct minimum where the cv reaches zero (cv = 0% @ 30 m swath width). The hump in the cv-swath width curve was higher than for triangles, but also stayed below the 15% line. As long as the ratio horizontal leg-spread width (3/12 in this case) remains limited, this is an excellent pattern, especially because the swath width can be higher than for triangles. There is a danger though that, due to changing material properties and environmental conditions, this pattern becomes M-shaped, which can make the pattern non-robust.

Trapezoid Wide (6/12) (fig. 6). Again, as in the triangular and narrow trapezoid cases, there is a distinct minimum in the cv-swath width curve where the cv reaches zero (cv = 0% @ 36 m swath width). The trapezoid is now so wide (the ratio horizontal leg-spread width is 6/12) that all swath widths between 22 and 32 m have cvs higher than 15%. This pattern has become non-robust, it is too "shouldered".

UNDESIRABLE SHAPES: SKEWED TRAPEZOID, CROWN, M-SHAPE, SKEWED M-SHAPE

The undesirable shapes are shown in figures 7 through 10.

Skewed Trapezoid (fig. 7). The skewed trapezoidal pattern is often seen when the spreader gate is not opening evenly. Also, side winds can introduce this kind of problem. As is clear from the cv-swath width curve, skewness introduces multiple humps with cv's higher than 15% from a swath width of 20 m and higher. Also, the cv seems to increase significantly overall. This pattern is non-robust and shows that skewness can be a major cause of producing low quality spread patterns.

Crown (fig. 8). The crown shaped pattern is often seen in practice when the fertilizer is not dropped evenly into the spreader ducts. It (like the Skewed Trapezoid pattern) produces multiple humps in the cv-swath width curve. This type of pattern is non-robust and should be avoided.

M-Shape (fig. 9). The M-shape is often seen in pendulum action spreaders and in aerial application when too much emphasis is placed on producing a wide swath width. It is slightly better than the Crown shape, but it is also non-robust and should be avoided.

Skewed M-Shape (fig. 10). The Skewed M-shaped pattern shows the effect of skewness on an already low quality pattern. Skewness seems to lift the cv-swath width curve (compare the M-shape to the Skewed M-shape and the Wide Trapezoid to the Skewed Trapezoid). As expected, this pattern is non-robust and should be avoided.

COMPARISON OF THE ROBUSTNESS OF HYPOTHETICAL SPREAD PATTERNS

In figure 11, the robustness factor AcvBF is shown for all shapes studied. The Back&Forth robustness factor was chosen because it represents the worst case scenario and since the Back&Forth flight mode is the most widely used technique in aerial application.

Using the 15% rule of thumb, the point where patterns tend to become non-robust is between the Trapezoid 4/12 (AcvBF = 4.48%) and the Trapezoid 5/12 (AcvBF = 6.18%). From this result a general rule of thumb can be stated: Patterns with robustness factors (AcvBF) lower than approximately 5% can be considered robust. The shapes that fall in this category are: Gaussian (AcvBF = 0.47%), Triangle Wide (AcvBF = 2.29%), Triangle Narrow (AcvBF = 2.55%), Trapezoid 3/12 (AcvBF = 3.01) and Trapezoid 4/12 (AcvBF = 4.48). The latter is referred to as the "critical trapezoid". All other patterns are non-robust and should be avoided.

CONCLUSIONS AND RECOMMENDATIONS

- Based on the results of analysis of 11 hypothetical overlapped spread patterns, the most robust shape found was a Gaussian.
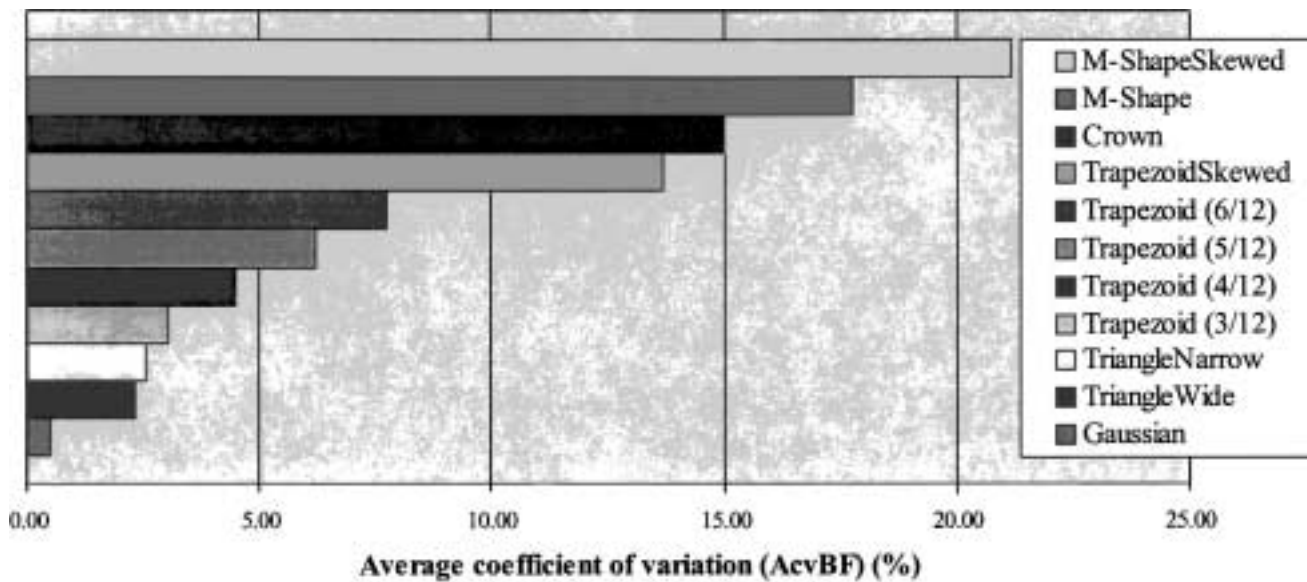
Figure 11–Quality measure for various hypothetical spread patterns.

- Triangles were found to be very robust shapes, especially when they are symmetric and have the target rate in the center.
- Trapezoids were found to be robust and efficient patterns, as long as they are not too "shouldered". The "critical trapezoid", which is on the limit of becoming non-robust, has a horizontal leg-spread width ratio of 4/12. Trapezoids are prone to transforming into M-shapes or skewed patterns (or a combination), which can make the pattern non-robust. Triangles are far less prone to this effect, which makes them the most desirable robust shapes. Further research could identify which factors are most dominant in the transformation process. Redesign of spreader equipment is one way to prevent M-shaped or skewed patterns, however, if material and environmental factors are dominant, the choice should be to strive for more triangular patterns which are inherently more robust.
- Skewness seems to increase the cv for all useful swath widths. There should be major efforts avoiding skewness especially in Back&Forth operation through proper testing and adjustments.

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